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THE FREE PISTON GAS GENERATOR

A VERIFICATION OF THE OPPENHEIM-LONDON DESIGN METHOD

L. A. WELGE

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by

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Academic Dean

ERRATA

Page vii	Change page number to "vi."
Page 1	Last line of paragraph one, change to read, "generator design method of significance".
Page 11	Second paragraph, change to read, "The subject method lends itself," etc.
Page 12	Seventh line, change to read, "that a 100 per cent error in estimating valve pressure drops will cause only fifteen per cent," etc.
Page 13	Last line, change to read, "cyclic speed at a desired or required value."
Page 15	Line seven, change to read, "designer will want to use higher pressures and temperatures and the engine problem will," etc.
Page 15	Third paragraph, third line, change "exceeded" to "reached."
Page 59	Footnote, change "page 62" to "page 60."

PREFACE

The free piston gas generator is a promising and newly developed prime mover. The potentialities of this new prime mover are such that extensive research and development studies are clearly warranted.

Thus the author, in considering a thesis in partial fulfillment of the requirements for a degree of master of science, saw an opportunity to pursue a stimulating and interesting study of a new thermodynamic device.

In assisting him to accomplish his aims the author wishes to express his appreciation for the guidance and suggestions of Professor A. L. London of the Mechanical Engineering Department at Stanford University.

The author also wishes to thank the staff and faculty of The United States Naval Post Graduate School at Monterey, California for technical assistance and aid in preparation of the thesis.

L. A. Welge

June 1952

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TABLE OF SYMBOLS AND ABBREVIATIONS

English Letter Symbols

A - piston (cylinder) cross-sectional area, ft^2

A/F - Air-fuel ratio in engine cylinder, $\frac{\text{lbs air}}{\text{lbs fuel}}$

a - acceleration, ft/sec^2

c_p - specific heat at constant pressure, Btu/lb°F

d - percentage valve pressure drop, percent of P_{abs}

D - cylinder bore; in, ft

E - energy, Btu/lb, Btu/$\sim$ or ft#/lb, ft#/$\sim$

f - percentage friction loss referred to the compressor cylinder work requirement

f_r - cycle frequency, $\sim$/min

F - force exerted by gas on piston, #

g - proportionality factor in Newton's Second Law (lb/#) (ft/sec^2)

h - enthalpy, Btu/lb

k - isentropic exponent, dimensionless

ℓ - piston length, in, ft

m - molecular mass, lb/mol

M - piston mass, lbs

n - polytropic exponent, dimensionless

P - pressure, $\#/in^2$abs

p - Pressure ratio, dimensionless

P_r - relative pressure function, dimensionless

Q - chemical energy to thermal energy conversion by combustion, Btu/lb

Q_{cw} - heat transferred to cooling water, Btu/lb

R - gas constant, 1545 $\frac{ft\#}{lb.mol^{o}R}$

r - volumetric compression ratio, dimensionless

s - stroke, piston displacement, in.

shp - shaft horsepower

T - temperature, ^{o}R or ^{o}F

Wk - work, Btu/lb, Btu/~ or ft#/lb, ft#/~

w - flow rate, lb/hr or lb/~

V - velocity, ft/sec

v - specific volume, ft^3/lb

x - distance from piston extreme position (IDC), in

Greek Letter Symbols

ϵ - ratio of "modified" engine expansion work to Otto cycle expansion work, %

ρ - density, lb/ft^3

η - efficiency, %

τ - piston travel time, sec.

Miscellaneous

lb denotes pounds mass in distinction to

# denotes pounds force

SUMMARY

It was desired to investigate the unique thermodynamic-dynamic features, the inherent characteristics, and the potentialities of a new type prime mover, the free piston gas generator. In particular it was desired to test and evaluate a recently proposed gas generator method of significance.

It was found that use of the proposed method led to accurate design results in a practical test and that the method was flexible and broad in possible application. The primary limitation of the method was the necessity of predicting thermodynamic properties of the engine cycle with good accuracy.

By investigating the inherent characteristics and potentialities of the prime mover, it was found that the most suitable use of the machine will be where heavy Diesel engines with complex clutching and transmission systems are presently used.

PREVIOUS INVESTIGATIONS

The present development of this interesting new prime mover, the free piston gas generator, has largely been due to the efforts of R. de Pescara and his associates. As a result, a free piston gas generator has been recently produced which has proven to be successful in actual application.

This is no mean achievement, since twenty-five years of research proved necessary at a cost of over a million dollars. One can realize how promising this machine must have been to have warranted such an investment.

Once the Pescara concern (S.I.G.M.A.) announced the production of an operational machine, considerable interest was arroused in the industry. A number of investigations were launched, several sponsored by departments of the United States government.

Unfortunately, several investigations in this country were aimed at developing high output machines for special purposes. These aims were not realized, and the free piston gas generator earned an undeserved reputation in certain quarters. However, the office of Naval Research sponsored a research project at Stanford University which resulted in the publication of the first full and significant report written to date in this country on the subject of free piston gas generators.[a]

In the above Office of Naval Research Report, the authors developed a free piston gas generator design method based on a thermodynamic-dynamic analysis of the Pescara type machine.

[a] Oppenheim and London (1)

In addition, the authors demonstrated the usefulness of affinity relations in design and analysis problems.

Reading from a paper[a] based on the above report, Professor A. K. Oppenheim presented the author's findings to a meeting of the A.S.M.E. at Seattle, Washington, in March of 1952.

Dr. G. Eichelberg of the Federal Institute of Technology, Zurich, Switzerland, has closely followed the Pescara development of the free piston gas generator. Dr. Eichelberg published a paper including the first comprehensive and authorative test results of a free piston gas generator ever published,[b] and it is therefore of considerable significance in the progress of development of this new prime mover.

A recent contribution to the literature is the work of R.E. Adams.[c] Unfortunately, this work is classified and cannot be discussed. It is hoped that at some later date the report will be released so that it may receive the wide consideration it deserves.

In addition, for a comprehensive background in the field of free piston machinery, a further number of works on associated subjects should be consulted.[d]

[a]Oppenheim and London (2)
[b]Eichelberg (3)
[c]Adams (13)
[d]Bibliography

OBJECTIVE

It was the objective of the thesis to test and evaluate a free piston gas generator thermodynamic-dynamic design method which was recently proposed in the literature.[a]

[a]Oppenheim and London (1)

EQUIPMENT

For the purpose of making a comparison between actual performance and the performance predicted by means of the method under test, a S.I.G.M.A. GS34 free piston gas generator and a hypothetical eighty-five percent efficient turbine were selected. Data and actual test results were available for the S.I.G.M.A. machine and it was representative of the present development of successful gas generators.

Various sectional and assembly views of the GS34 model gas generator may be seen in Appendix A.

PROCEDURE

Characteristics found by trial of an actual machine were compared to the predicted characteristics determined by means of the subject method and the use of only those data normally independent in the design problem.

DISCUSSION

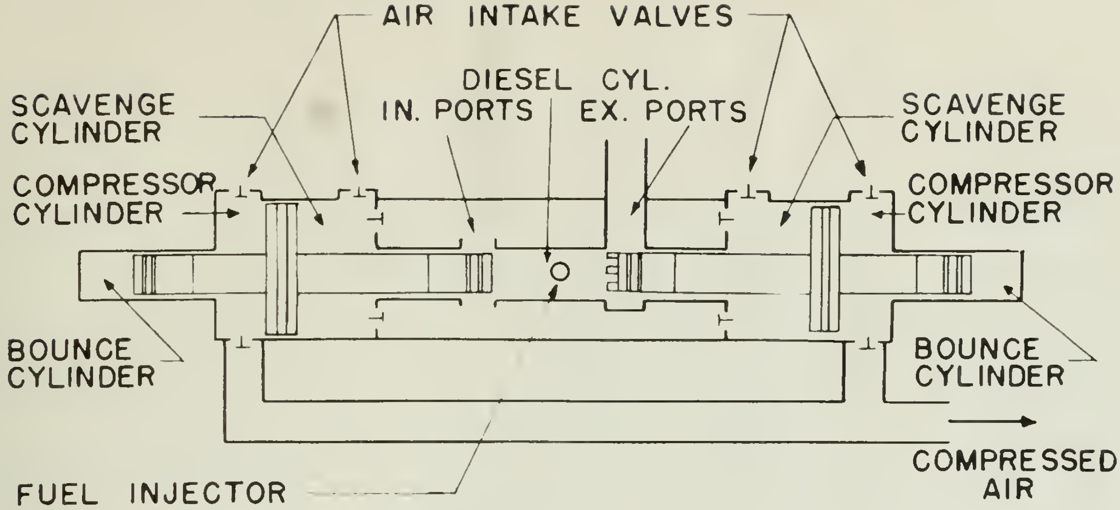
INTRODUCTION

The symmetrical free piston system consists of an opposed piston uniflow two stroke cycle Diesel engine which provides the energy necessary to drive associated reciprocating compressors.

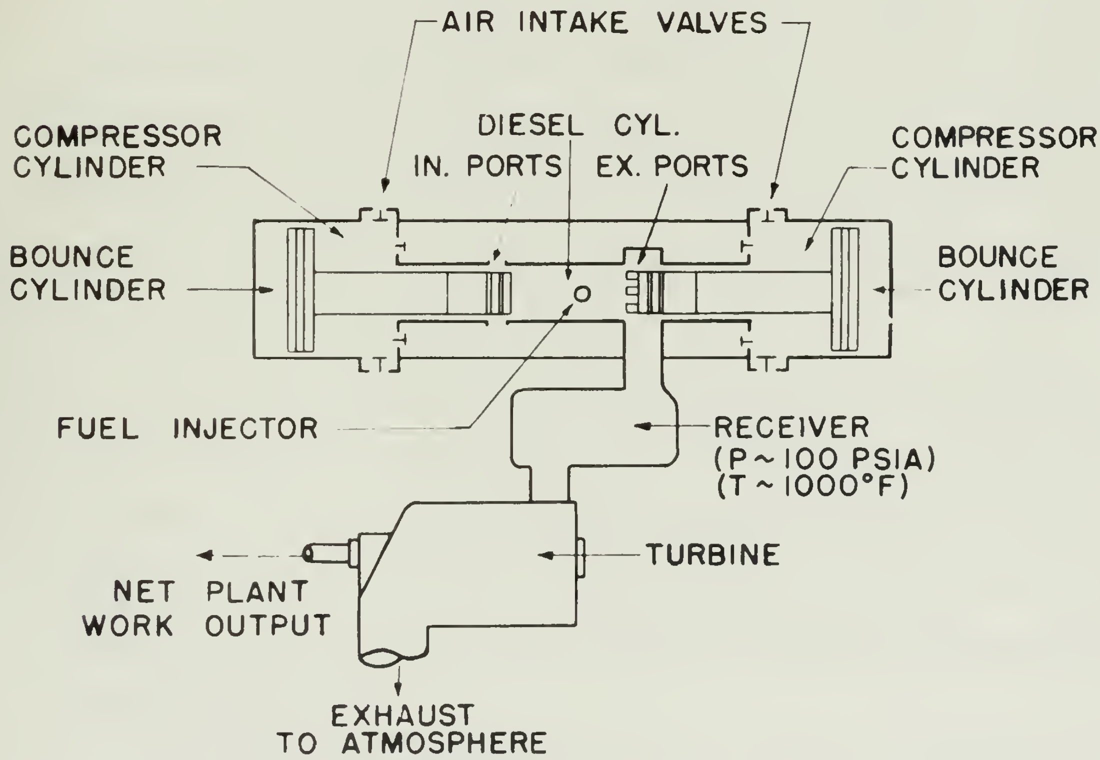
In contrast to conventional reciprocating machinery no linkages couple the engine and the compressor. In fact, apart from piston phasing control linkages the main moving parts - the two pistons - are independent of mechanical restraints and from this characteristic is derived the name of the system.

The free piston system has two fundamental applications. As a free piston compressor, the useful output of the machine is compressed air. As a free piston gas generator, the useful output is high temperature compressed gas for use in a non-condensing turbine.

For the compressed air application the engine is provided only with sufficient compressed air for scavenging, and the engine exhausts at atmosphere pressure. For the gas generator application, the engine operates at the high supercharge of full compressor output pressure. The engine exhausts to a turbine at only a slightly lower pressure. All compressed air passes through the engine as a combustion medium, a coolant or a scavenging agent. The two applications are shown schematically in figure one.



(a). FREE - PISTON COMPRESSOR SYSTEM.



(b). FREE - PISTON GAS GENERATOR-TURBINE SYSTEM.

FIG. 1

Because of its inherent characteristics, its potentialities and its unique thermodynamic-dynamic features the gas generator application is particularly interesting. The inherent characteristics of the gas generator are considered in Appendix B, the potential applications of the gas generator in Appendix C, and a thermodynamic-dynamic analysis in Appendix D.

PURPOSE OF THESIS

There is a growing industrial interest in the gas generator with several companies actively engaged in research or production in this country and abroad. But the fundamental design and analysis problem has not been previously considered in the literature. Because the problem is unique and complex it may be presumed the lack of such information limited practical development and exploitation of the gas generator. Thus the recently proposed Oppenheim-London design method was a significant contribution.

The proposed design method was developed as part of a naval research project. In the report of findings the method was verified by comparison of actual gas generator characteristics and characteristics predicted by means of the design method. But numerous machine data were assumed and thus the verification was less conclusive than might be desired.

Accordingly, the purpose of the thesis was to test and to evaluate the proposed design method.

PROCEDURE

The S.I.G.M.A. GS34 model gas generator was selected for the test vehicle since all important data[a] and actual trial results[b] were available for this machine, and it was typical of the present gas generators in production.

In Europe it is customary to consider the isentropic power of such devices as the gas generator. Thus the results of the test on the actual generator made in France were given on the basis that an isentropic turbine was part of the gas generator system.[c] In this country this is not a common procedure and some confusion might result, so the results of the actual gas generator trial as given were modified to include a turbine which had an adiabatic efficiency of eighty-five percent. The actual gas generator trial results as modified were then compiled and tabulated.

The characteristics of the gas generator turbine system were then predicted by means of the method under test using only the actual machine data which would be normally independent. The predicted results were compiled and tabulated for ready comparison with the actual engine trial results.

The intake air temperature and pressure existing at the time of the machine trial were not known. Also several critical engine thermodynamic data had to be predicted, a requirement inherent to the subject design method. The data which required prediction or assumption are considered in Appendix A.[d]

[a]Table of Data
[b]Table of Results

[c]Eichelberger (3)
[d]A general discussion may be found in Oppenheim-London (1)

FINDINGS

When the known trial results and the results predicted analytically by means of the subject method were compared it was found that there was no significant disagreement between comparable items.[a]

It may not be concluded that in all cases the method in question will yield precise results without modifications in idealizations. But it is certain that good results may be expected from design of a machine which is not radically different in character from the S.I.G.M.A. GS34 machine.

EVALUATION OF METHOD

The subject lends itself to ready solution of design and analysis problems to be found in the field, as the assumptions and idealizations may be readily modified at the discretion of the designer to suit requirements. With the affinity relations incorporated[b] the subject method is further broadened in possible application.

However, the method has certain limitations. Design results are sensitive to the thermodynamic conditions specified to exist in the gas generator engine. In predicting these conditions advantage may be taken of experience in the field of supercharged uniflow opposed piston Diesel engines. However a modified Otto cycle is specified, and the correspondence between conditions in a Diesel engine and a gas generator engine is not entirely direct. Thus a certain facility is necessary to accurately predict the thermodynamic conditions in the gas generator engine. But this requirement will not always pertain, for as test data

[a]Maximum discrepancy was 1.8%

[b]Appendix D

of various gas generators are accumulated and correlated, an empirical method of prediction of thermodynamic engine conditions will be practical and convenient.

In contrast, results are not sensitive to poor estimates of valve pressure drops, frictional characteristics, or to be idealizations concerning the gas medium. For example, it has been shown in one case[a] that doubled valve pressure drops caused fifteen percent changes in engine piston area and less than eleven percent changes in other variables effected at all.

In problems in which affinity relations are employed, it is to be remembered that the validity of results is predicated on a constancy of thermodynamic conditions. Thus if it were desired to design a gas generator of a partial horsepower rating from data of the 1000 horsepower S.I.G.M.A. GS34 model, there would be little guarantee that by using affinity relations satisfactory results could be achieved; it is certain that the thermodynamic conditions would differ materially in the two engines.

In solving a design problem using the subject design method, certain results are not determined without considerable and tedious calculation; notable are the calculations involved in finding the cyclic frequency.

If a loss of accuracy may be tolerated, the calculation of cyclic frequency may be greatly simplified by assuming that net piston forces may be represented by equivalent linear spring type forces. The designer will be able to tolerate some inaccuracy in the prediction of

[a]Oppenheim and London (1)

cyclic frequency provided that in the actual machine the weight of the pistons may be modified within sufficiently broad limits to fix the cyclic speed at desired values.

CONCLUSIONS

A. The Oppenheim-London gas generator design method was conclusively verified by test.

B. With affinity relations incorporated, the design method is flexible and broad in application, adaptable to design and analysis problems to be encountered in the field.

C. Design results are sensitive to the thermodynamic conditions specified to exist in the gas generator engine, but not to estimates of valve pressure drops, frictional characteristics, or to idealization of the properties of the gas medium.

RECOMMENDATIONS

Because of the machine's important potentialities[a] and because little essential experimental research has been attempted to date, the following work is suggested.

ENGINE WEAR RESEARCH - Research on gas generator engine wear would be valuable. Whatever ingenuity is practiced in design, as gas generators are desired to be more efficient and powerful, the engine wear problem will remain a primary consideration. In this regard, since the piston is essentially a projectile, applicable findings of the many investigations in the field of interior ballistics might prove of value.

RESEARCH ON FUELS - Study of fuels and injection techniques would be rewarding. Faster burning is desirable, provided destructive loadings and temperatures are not exceeded. In addition cheaper fuels are desirable, with evidence already supporting a successful utilization.

STUDY OF PHYSICAL ARRANGEMENTS - Study and trial of various promising physical arrangements and modifications would be rewarding. For example, the elimination of the separate bounce cylinder has been proposed, the machine to operate on the principles of the Junkers type free piston compressor.

STUDY OF NON-LINEAR BOUNCE SYSTEMS - Provided stabilization is possible, a bounce system with radically non linear spring characteristics would result in relief of peak temperatures and pressures

[a]Appendix B and Appendix C

more rapidly and could support higher cyclic speeds. An appropriately ported bounce cylinder which results in a rapid build up of bounce pressures only at the end of the piston stroke is an example of the possible arrangements. Study and experiment might well indicate the feasibility of such systems.

BIBLIOGRAPHY

1. Oppenheim, A.K. and London, A.L.: The Free Piston Gas Generator, A Thermodynamic-Dynamic Analyses. Technical Report FP-1 Navy Contract N6-onr-251 Task Order 6 (NR-035-104) Department of Mechanical Engineering Stanford University February 15, 1950.

2. Oppenheim, A.K. and London, A.L.: Free Piston Engine Development - Present Status and design aspects. Paper 52-S-17, Advance Copy, ASME Meeting, Seattle, Washington, March 24, 1952.

3. Eichelberg, G.: Free-Piston Generators, "Lé Genie Civil" and "Schweizerische Bauzeitung", p. 661-667, November 27 and p. 673-679, December 4, 1948. Review: Power, Vol. 93, No. 1, p. 79-81 and 156-158, January, 1949.

4. Neumann, K.: Junkers Free Piston Compressor, Zeitschrift des Vereins Deutscher Ingenieure, Vol. 79, No. 6, p. 155-160, February 1935. Translation: NavShips (330) Research Memo No. 4-44, Sept. 1944.

5. Pescara, R. de: Free-Piston Engines, Journal de la Sociétè des Ingenieurs de l' Automobile, Vol. 10, No. 7, p. 423-233, 1937. Review: Automobile Industries, Vol. 78, No. 6, p. 181-182, and 184, 1938: Gas and Oil Power, p. 229-233, Sept. 1938. Translation: NavShips (330) Research Memo No. 4-44 Sept. 1944.

6. Pescara, R. de: Combustion in Chambers of variable Volumes, Chaleur et Industrie, Vol. 20, p. 143-150, 1939. Translation: NavShips (330) Research Memo No. 4-44 Sept. 1944.

7. Pescara, R. de: The Free-Piston Gas Generator and Turbine, Chaleur et Industrie, Vol. 20, p. 211-214, 1939. Translation: NavShips (330) Research Memo No. 4-44 Sept. 1944.

8. Sales Brochures (a) Pescara Free-Piston Machines, Sociêtê Gênerale des Constructions Electriques and Mechaniques (Als-Thom), 38 Avenue Kleber, Paris. Translation: NavShips (330) Research Memo No. 4-44 Sept. 1944.

(b) Free-Piston Diesel Air Compressors, Pescara Patents, Sociêtê Industrielle Genêrale de Mechanique Appliquêe, S.I.G.M.A., 61 Av. Franklin D. Roosevelt, Paris (8e)

9. Oederlin, F.: The Supercharging of Two Stroke Diesel Engines, Sulzer Technical Review, p. 1-21, Dec. 1941 and Mechanical Engineering, p. 779-783, Nov. 1942. Review: Gas and Oil Power, p. 113-120, July 1942 and Power, p. 102-103, Dec. 1942.

10. Farmer, H.O.: Free-Piston Compressor Engines, Proceedings of the Institution of Mechanical Engineers, Vol. 156, No. 3, p. 253-271, 1947.

11. Irwin, P.L.: Free-Piston Gas Generator, Baldwin Locomotive Works (Nobs 1455) U 4104, Progress Report No. 53, Dec. 1948.

12. Small, L.F.: Free Piston Gas Generator Brightens Gas Turbine Future, Paper read before Diesel Symposium of Diesel Engine Manufacturers Association, at Cornell University, September 3, 1948 and given before the S.A.E. Summer Meetings, June, 1949; Journal of the American Society of Naval Engineers, Vol. 61, No. 1, p. 23-34, February 1949; Diesel Progress, p. 30-32, Sept. 1949. S.A.E. Journal, Vol. 157, No. 10. p. 62-63, Oct. 1949

13. Adams, R.L.: Analyses and Evaluation of the Free-Piston Gas Generator. ONR Contract N5-onr-D7640, Division of Applied Science, Harvard University, Cambridge, Mass. RESTRICTED Classification.

APPENDIX A

TABLE OF PREDICTED AND ACTUAL TEST RESULTS
S.I.G.M.A. GS34 GAS GENERATORS

Item	Units	Predicted Results	Actual[a] Results
Gas delivery Temperature to turbine	deg. R	1416	1405
Gas delivery pressure to turbine	p.s.i.a.	64.5	64.5
Net power output from turbine	hp.	1138	1138
Gas flow rate	lbs/hr.	28900	29000
Fuel rate	lbs/hr.	456	460
Plant thermal efficiency	%	35.0	34.5
Engine indicated thermal efficiency	%	29.8	29.8
Turbine adiabatic efficiency	%	85.0	85.0
Engine cycle thermal efficiency (Otto cycle basis)	%	47.4	47.4
Cooling system losses	%	19.4	19.4
Engine cyclic rate	~/min.	613	613
Engine-compressor air weight ratio	-	0.514	0.516
Engine air-fuel weight ratio	-	32	32
Specific fuel consumption	lbs/sh hp.hr.	0.401	0.404
Air delivered per cycle	lbs.	0.386	0.388

[a]From reference three adjusted for turbine performance.

TABLE OF S.I.G.M.A. GS34 GAS GENERATOR DATA

Item	Units	Used in Analyses	Actual
Top engine cycle temperature	deg. R	2340	-[a]
Gas inlet temperature to compressor	deg. R	540	-
Bounce cylinder gas mean temperature	deg. R	500	500[b]
Engine top gas pressure	p.s.i.a.	1091	1081[c]
Bounce cylinder gas pressure at end of expansion	p.s.i.a.	26.45	26.45[b]
Compressor inlet gas pressure	p.s.i.a.	14.7	-
Valve effective pressure losses	%	5.0	-
Piston solidity	%	24.5	24.1[b]
Friction work loss per stroke, including auxiliary load	%	6.0	6.0[b]
Expansion stroke efficiency	%	77.4	-
Friction heat loss to cooling water, percent of total frictional losses	%	47.0	-
Compressor piston clearance	%	14.0	14.0[d]
Total piston length	ins.	49.0	49.0
Engine Piston length	ins.	38.0	38.0

[a]Items not reported or otherwise available.
[b]From manufacturer courtesy Professor A.L. London

[c]Not reported whether absolute or gauge pressure.
[d]Scaled from plan with allowance for valve recess volumes.

TABLE OF S.I.G.M.A. GS34 GAS GENERATOR DATA (Cont.)

Item	Units	Used in Analyses	Actual
Compressor piston length	ins.	11.0	11.0
Engine effective stroke	ins.	9.7	9.7[a]
Piston stroke at test conditions	ins.	17.5	17.5
Piston weight	lbs.	1110	1110[a]
Compressor-engine piston area ratio	-	5.86	5.86
Engine-compressor air weight ratio	-	0.514	0.516
Engine piston area	sq. ft.	0.982	0.979

[a]From manufacturer courtesy Professor A.L. London

EVALUATION OF UNKNOWN DATA FOR S.I.G.M.A.
GS34 GAS GENERATOR TEST

GAS INTAKE TEMPERATURE - The machine was tested in March, near Lyon, France. The value of temperature was assumed to be 540 degrees Rankine. The effects of error within possible limits are minor.

GAS INTAKE PRESSURE - The machine was tested near sea level. The pressure was assumed to be 14.7 p.s.i.a. The possible error will not effect results.

VALVE PRESSURE DROPS - Taking advantage of experience in the field of compressor design, the valve pressure drops were assumed to be five percent. It was learned from conversation that the manufacturer found seven percent pressure drops in actual test. Such a difference does not effect results to any extent[a].

TOP CYCLE TEMPERATURE - In appraising the value recommended previously[a], advantage was taken of experience in the field of supercharged uniflow diesel engines. Due consideration was given to the fact that the initial phase of combustion is extended because of the more rapid expansion stroke, and a modified Otto cycle was predicated in the case of the gas generator analysis.

It is felt that the value assumed, 2340 degrees Rankine, is accurate to 500 degrees. It is to be remembered this temperature is an effective one consistent with the idealization of a modified Otto cycle approximating the actual engine processes. While a critical item, an error of 250 degrees in the top cycle temperature does not effect results critically[a].

[a]Oppenheim and London (1)

EXPANSION STROKE EFFICIENCY - From considerations of a similar nature to those outlined on the preceding page, the previously recommended value[a] of 77.4 percent was accepted.

FRICTION HEAT LOSS TO COOLING WATER - This item is concerned in the heat balance. The heat balance was of little interest in the Oppenheim-London method so the friction heat loss to the cooling was determined as 47 percent by inverting the problem. Normally, the frictional loss to coolant would be predicted by usual means, and then by the heat balance the total coolant heat loss could be determined.

[a]Oppenheim and London (1)

DESIGN CALCULATIONS BY THE OPPENHEIM-LONDON DESIGN METHOD

I. COMPRESSOR CHARACTERISTICS (per 1 lb. of air delivered/cyl. x cycle)

A. Basic Idealizations

1. Equation of State of air:

   Perfect Gas with const. specific heat capacities.

2. Leakage: zero

3. Polytropic exponent for air compression and expansion: $n = 1.35$

4. Valve pressure drops: for inlet: $d_o = 5\%$ of P_o

   for discharge: $d_D = 5\%$ of P_3

B. Independent variables

1. Pressure ratio: $P_D/P_O = 4.85$

2. Stroke length: $s_s = 17.5$ ins.

3. Clearance: $c = 14\%$ $S_c = 2.446$ ins.

4. Intake state: $P_o = 14.7$ #/in² abs.; $T_O = 540°R$

5. Delivery state: $P_D = 71.3$ #/in² abs.

C. Basic Relations

1. Thermodynamic properties of air:

   $\frac{R}{m} = 53.3$ ft #/lb °R and $k = 1.40$

D. Dependent variables

1. Pressure at state (1) & (4):

   $P_1 = P_4 = (1-d_o)\ P_o = 13.96$ p.s.i.a.

2. Temperature at state (1) & (4):

   $T_1 = T_4 = T_o = 540$ deg. R

3. Sp. volume at state (1) & (4):

   $v_1 = v_4 = \frac{R}{m}\ \frac{T_1}{P_1} = 0.37 \frac{(2)}{(1)} = 14.3$ ft³/lb

4. Pressure at state 2 & 3 :

$$P_2 = P_3 = \frac{P_D}{1 - d_D} = 75.1 \qquad \text{p.s.i.a.}$$

5. Specific volume at state 2 & 3:

$$v_2 = v_3 = v_1 \left(\frac{P_1}{P_2}\right)^{\frac{1}{n}} = 3 \left[\frac{1}{4}\right]^{\frac{1}{1.35}} = 4.11 \qquad ft^3/lb$$

6. Temperature at state $\boxed{2}$ & $\boxed{3}$:

$$T_2 = T_3 = \frac{P_2\, v_2}{\frac{R}{m}} = \frac{4 \times 5}{0.370} = 835 \qquad \text{deg. R}$$

7. Stroke position at state 1:

$$s_1 = s_c + s_s = 19.946 \qquad \text{ins.}$$

8. Stroke position at state 2:

$$s_2 = s_1 \frac{v_2}{v_1} = 7 \times \frac{5}{3} = 5.73 \qquad \text{ins.}$$

9. Stroke position at state 3 :

$$s_3 = sc = 2.446 \text{ ins.} \qquad \text{ins.}$$

10. Stroke position at state 4:

$$s_4 = s_3 \frac{v_4}{v_3} = 9 \times \frac{3}{5} = 8.53 \qquad \text{ins.}$$

11. Piston Area:

$$A_c = \frac{v_1}{s_1 - s_4} = \frac{1728 \; 3}{7 - 10} = 2160 \qquad in^2/\text{lb air del.}$$

12. Coordinates of the Indicator card and the force-stroke diagram:

$$Pv^n = P_1 v_1^{1.35} = 1 \times 3^{1.35} = 506.9 \qquad ft^3\#/in^2 \text{ lb air}$$

	a	b	c	d	e	f
	P	$v^{1.35}$	v	Piston position compression stroke	Piston position expansion stroke	Force = A_cP
	#/in^{2}abs	ft^3/lb	ft^3/lb	$s = s_1 \times \frac{v}{v_1}$ ins.	$s = s_3 \frac{v}{v_3}$ ins.	10^3 #
		$\frac{12}{a}$	$b^{1/1.35}$	$\frac{7}{3} \times c$	$\frac{9}{5} = c$	11 x a
1,4 State	13.96	36.30	14.30	19.95	8.53	30.2
	20	25.35	10.95	15.26	6.53	43.2
	30	16.90	8.13	11.35	4.85	64.8
	40	12.67	6.57	9.17	3.92	86.4
	50	10.14	5.57	7.77	3.32	108.1
	60	8.45	4.86	6.78	2.89	129.9
	70	7.24	4.34	6.06	2.58	151.4
2,3 State	75.1	6.76	4.12	5.75	2.45	162.3

13. Work of polytropic process 1 - 2 and 3 - 4 per lb. of air in cylinder:

$$W_{12} = W_{34} = \frac{1}{n-1}(P_2v_2 - P_1v_1) = \frac{144}{0.35 \times 778} \left(4 \times 5 - 1 \times 3\right) =$$

57.3 Btu/lb air

14. Mass of air for process 1 - 2

$$w_{12} = \frac{s_1}{s_1 - s_4} = \frac{7}{7 - 10} = 1.745 \quad \text{lb/lb air del.}$$

15. Mass of air for process 3 - 4:

$$w_{34} = \frac{s_3}{s_2 - s_3} = \frac{9}{8 - 9} = 14 - 1 = 0.745 \quad \text{lb/lb air del.}$$

16. Work of exhaust process $\boxed{2}$ - $\boxed{3}$:

$$W_{23} = F_2(s_2 - s_3) = f_2 \frac{8 - 9}{12 \times 778} = 57.3 \quad \text{Btu/lb air del.}$$

17. Work of intake process 4 - 1 :

$W_{41} = F_1(s_1 - s_4) = f_1 \dfrac{7 - 10}{12x778} = 36.9$ BTU/lb air del.

18. Work of compression stroke 1 - 2 - 3 :

$W_{compr.} = w_{12}W_{12} + W_{23} = 14 \times 13 + 16 = 157.5$ BTU/lb air del.

19. Work of expansion stroke: 3 - 4 - 1 :

$W_{exp.} = w_{34}W_{34} + W_{41} = 15 \times 13 + 17 = 79.6$ BTU/lb air del.

20. Compressor cycle work:

$W_c = W_{compr.} - W_{exp} = 18 - 19 = 77.85$ BTU/lb air del.

CHECK:

$Wc = \dfrac{n}{n-1}(P_2v_2 - P_1v_1) = 1.35 \times 13 = 77.5$ BTU/lb air del.

II. ENGINE CHARACTERISTICS (per 1 lb. of air del./cyl. x cycle)

A. Basic Idealizations

1. Equation of state of air: Perfect Gas with const. sp. heat capacities

2. Basic Otto cycle: "Effective" isentropic exponent $k = 1.30$ and a Q input corresponding to fuel: air ratio x LHV of fuel.

3. Engine cycle: modified Otto cycle defined as follows:

   a) compression stroke the same as for Otto cycle.

   b) thermal energy input the same as for the Otto cycle of (a)

   c) Expansion work equal to 77.4% of the Otto cycle exp. work

   d) Max. temperature fixed at 2340° R.

   e) expansion stroke: a polytrope with a magnitude of n satisfying condition (c).

4. Friction work per cycle: 12% of compressor work.

5. Valve pressure drops: d = 5% of P for each port passage

6. Engine intake temp. equals comp. exhaust temp.

B. Independent variables

1. Air-fuel ratio: $f = 32 \div 1$

2. "Effective" stroke: $s_e = 9.7$ in.

3. Compression ratio: $r_c = 8.5 \div 1$

4. Lower heating value of fuel: LHV = 18,200 Btu/lb.

C. Basic Relations

1. Thermodynamic properties

$$\frac{R}{m} = 53.3 \text{ ft \#/lb}^\circ\text{R}; \quad c_r = \frac{\frac{R}{m}}{k-1}, \quad k = 1.30$$

2. $W_{k \text{ eng net}} = W_{k \text{ compr net}} + W_{k \text{ friction total}}$

D. Dependent variables

1. Pressure at state 1:

$$P_1 = \text{I.B.5}\,(1-d) = 67.7 \qquad \text{p.s.i.a.}$$

2. Temperature at state 1:

$$T_1 = T_{3\,comp} = \text{I.D.6} = 835 \qquad \text{deg. R}$$

3. Sp. volume at state 1 & 4:

$$v_1 = \frac{R}{m}\,\frac{T_1}{P_1} = \frac{C1}{144}\,\frac{2}{1} = 4.57 \qquad ft^3/lb$$

4. Pressure at state 2:

$$P_2 = P_1\, r_c^{\,k} = 1 \times B3^{A2} = 1091 \qquad \text{p.s.i.a.}$$

5. Specific volume at state 2 & 3:

$$v_2 = v_3 = \frac{v_1}{r_c} = \frac{3}{B3} = 0.5375 \qquad ft^3/lb$$

6. Temperature at state 2:

$$T_2 = \frac{P_2 v_2}{R/m} = \frac{144\,(4) \times (5)}{C1} = 1584 \qquad \text{deg. R}$$

7. Specific heat capacity at const. volume:

$$c_v = \frac{R/m}{k-1} = \frac{C1}{(A2-1)\times 778} = 0.2284 \qquad \text{BTU/lb, deg. R}$$

8. Thermal energy addition for Otto cycle:

$$q = \frac{LHV}{f} = \frac{B4}{B1} = 568.0 \qquad \text{BTU/lb, eng. air}$$

9. Temperature at state 3:

$$T_3 = T_2 + \frac{q}{c_v} = 6 + \frac{8}{7} = 4070 \qquad \text{deg. R}$$

10. Thermal efficiency of Otto cycle:

$$e_{Otto} = 1 - \frac{1}{r_c^{\,k-1}} = 1 - \frac{1}{B3^{A2-1}} = 0.474$$

11. Work of compression:

$$Wk_{1\cdot 2} = c_v T_2 \times e_{Otto} = 7 \times 10 \times 6 = 171.5 \qquad \text{BTU/Lb eng air.}$$

12. Work of expansion:

$$Wk_{3'4'} = e_{ex}C_vT_3e_{Otto} = A.3 \times 7 \times 9 \times 10 = 341 \quad \text{BTU/lb eng. air.}$$

13. Engine net work:

$$W'_{eng} = W_{eng\ exp} - W_{eng\ compr} = 12 - 11 = 169.5 \quad \text{BTU/lb eng. air}$$

14. Ratio of eng. air/compressed air delivered:

$$W = (1 + fr) \frac{W_{compr\ net}}{W_{eng}} = \frac{(1 + A4)\ I.D.20}{13}$$

$$= 0.514 \qquad \text{lb. eng.air/lb air del}$$

15. Piston Area:

$$A = \frac{w_{eng}\ xv_1}{\text{eff. stroke \& clearance}} = \frac{w_{eng}\ xv_1}{s_e \times \frac{(r_c)}{(rc-1)}} = \frac{12(B3 - 1)}{B2 \times B3}$$

$$14 \times 3 = 2.56 \qquad \text{ft}^2\text{/lb air del.}$$

16. Polytropic exponent of expansion (modified cycle):

$$W_{exp} = \frac{R/m}{n-1}\ T_{3'} \left\{1 - \frac{1}{r_c^{\,n-1}}\right\} = \frac{C1 \times A3}{(n-1)\ 778} \left\{1 - \frac{1}{B3^{\,n-1}}\right\}$$

$$W_{exp} = 160.4 \left(\frac{1}{n-1}\right)\left(1 - \frac{1}{8.5^{(n-1)}}\right) \qquad \text{BTU/\# eng air.}$$

a	b	c	d	e	f
n	$n-1$	$r_c^{\,n-1}$	$\frac{1}{r_c^{\,n-1}}$	$1-\frac{1}{r_c^{\,n-1}}$	W_{exp}
	a - 1	r_c b	$\frac{1}{c}$	1 - d	$\frac{160.4\ e}{b}$
1.0050	0.005000	1.010758	0.989357	0.010643	341.43
1.0070	0.007000	1.015093	0.985135	0.014865	340.62

From curve $n = F\left[W_{exp}\right]$[a] for $W_{exp} = 12 = 341$ BTU/lb eng air.

17. Pressure at state 3':

$$P_{3'} = \frac{R\ T_{3'}}{m\ v_3} = \frac{C1\ A.3}{144\ \ 5} = 1611 \qquad \text{p.s.i.a.}$$

18. I.T.C. piston position:

$$s_c = \frac{s_e}{r_c - 1} = \frac{B2}{B3 - 1} = 1.293 \qquad \text{ins.}$$

[a] Appendix A

.19 Coordinates of the indicator card and the force stroke diagram.

Line No.	a		b	c	d	e	f	g	h
	Piston Position		(s/s_c)	$(s/s_c)^k$	$(s/s_c)^n$	$P_{Compr.\ str.}$ $\#/in^2$	$P_{exp.\ str.}$ $\#/in^2$	$F_{Compr.\ Str.}$ $10^3\#/lb$ air del.	$F_{exp\ str.}$ $10^3 lb$ air del.
	ins. from ℄		$\frac{a}{18}$	$b^{1.3}$	$b^{1.006}$	$1 \frac{c_2}{c}$	$\frac{17}{d}$	$\frac{144}{1000}$ 15 x e	15 x f x $\frac{144}{1000}$
1.	IB2 + 18	18.793						24.94	24.94
2.	B2 + 18	10.993	8.50	16.15	8.60	67.7	187.5	24.94	69.1
3.		7.000	5.41	8.96	5.46	122.0	259.3	44.95	108.9
4.		5.000	3.87	5.81	3.90	188.2	412.3	69.4	151.8
5.		4.000	3.09	4.34	3.11	252.0	518.0	92.9	191.1
6.		3.000	2.32	2.986	2.33	365.4	692	134.6	255.0
7.		2.000	1.546	1.762	1.549	621	1063	229.0	392.0
8.		1.500	1.160	1.213	1.161	902	1387	332.2	512.0
9.	18	1.293	1.000	1.000	1.000	1093	1611	403.0	594.7

20. Work of compression stroke:

$$W_{eng\ compr} = W_{eng}\ W_{k12} + F_1(s_6 - s_1)$$

$$= 14 \times 11 + \frac{19\ gl}{778 \times 12}\left[I.B.2 - B.2 \right]$$

$$= 109.1 \qquad \text{Btu/lb air del.}$$

21. Work of expansion stroke:

$$W_{eng\ esp} = W_{k3'4} + F_1\ (s_6 - s_1) -$$

$$= 14 \times 12 + \frac{19\ h_1}{778 \times 12}\left[I.B.2 - B.2 \right]$$

$$= 196.3 \qquad \text{Btu/lb air del.}$$

21. (a) Net work:

$$\Delta W = 21 - 20 = 87.2 \qquad \text{Btu/lb air del.}$$

Check:

$$\Delta W = \left[1 + A.4 \right] \times I.D.20 = 87.2 \qquad \text{Btu/lb air del.}$$

22. Indicated thermal efficiency

$$e_t = \frac{W_{k\ eng}}{W_{air\ eng} \times q} = \frac{21 - 20}{14 \times 8} = 0.298$$

23. Indicated engine efficiency:

$$e_{eng} = \frac{22}{10} = 0.628$$

III. BOUNCE CYLINDER CHARACTERISTICS (per 1 lb of air del/cyl. cycle)

A. Basic Idealizations

1. Equation of state of air: Perfect gas with const. sp. heat capacities

2. Leakage: zero

3. Polytropic exponent for air compression and expansion: $n = 1.40$

4. Friction work fraction per stroke $= \frac{1}{2}$ (Friction work fraction per cycle) $= 0.06$

5. W_k bounce compr $= W_k$ bounce exp. and
   $F(x)$ bounce compr. $= F(x)$ bounce exp.

B. Independent variables

1. State at start of compression:

a. $p_1 = 26.46$		p.s.i.a.
b. $T_1 = 500$		^{o}R
c. $S_s = 17.5$		in

2. Piston area relative to compressor cylinder:[a]

$$1.008\ A_B = 1.021\ A_c + A_e = 1.021\ 9\ (ID11) + (II\ D15)$$
$$A_B = 2557 \quad in^2$$

C. Basic Relations

1. Thermodynamic properties of air:

$$\frac{R}{m} = 53.3 \text{ ft \#/lb } ^{o}R; \quad k = 1.40$$

2. W_k bounce compr $= W_k$ bounce exp.

$= W_k$ eng exp stroke $+ W_k$ compr exp stroke

$- W_k$ fr per stroke

$= W_k$ eng compr stroke $+ W_k$ compr compr stroke

$+ W_k$ fr per stroke

[a] The usual $A_B = A_{compr} + A_{eng}$ is modified by internal ductwork in the S.I.G.M.A. GS34 model

D. Dependent variables

1. Specific volume at state 1 :

$$v_1 = \frac{R}{m} \frac{T_1}{P_1} = \frac{C1}{144} \times \frac{B1}{B1} = 6.99 \qquad ft^3/lb$$

2. Work required per stroke

$$W_k \text{ bounce compr.} = W_k\text{eng. exp} + W_k \text{ compr. exp.} - W_k\text{fr. per stroke}$$

$$= II.D.21 + I.D.19 - A4 \times I.D.20 =$$

$$= 271.2 \qquad \text{Btu/lb air del. cycle}$$

Check:

$$W_k \text{ bounce exp} = W_k \text{ eng compr stroke} + W_k \text{ compr compr stroke} + W_k \text{ fr per stroke} =$$

$$= II.D.20 + I.D.18 + I.D.20 \times A4$$

$$= 271.3 \qquad \text{Btu/lb air del. cycle}$$

3. Compression ratio and ratio of bounce air/compressed air delivered

a. $$\frac{W_k \text{ bounce}}{M_{\text{bounce air}}} = \frac{R}{m} \frac{T_1}{n-1} (r_{cb}^{\,n-1} - 1) = \frac{C1}{778} \times \frac{B1}{A3-1} (r_{cb}^{\,A3-1} - 1)$$

$$= 85.75(r_{cb}^{\,0.4} - 1) \qquad \text{Btu/lb bounce air}$$

b. $$M_{\text{bounce air}} = \frac{A_{\text{bounce}}}{v_1} (\text{Stroke} + \text{clearance}) = \frac{A_b}{v_1} \left(\frac{str + cl}{cl}\right) \cdot \left(\frac{cl}{str}\right) \cdot str =$$

$$= \frac{A_b}{v_1} r_{cb} \times \frac{1}{r_{cb}-1} \times s_c = \frac{B.2}{1} \frac{r_{cb}}{r_{cb}-1} \; I.B.2$$

$$= 3.71 \left(\frac{r_{cb}}{r_{cb}-1}\right) \qquad \text{lb/lb air del.}$$

a	b	c	d	e	f
r_{cb}	$\frac{r_{cb}}{r_{cb}-1}$	r_{cb}^{n-1}	$M_{b.air}$ $\frac{lb}{lb\ air\ del}$	$\frac{W_k}{M_{b.air}}$ $\frac{BTu}{lb\ b.air}$	$W_{k_{b.}}$/lb air del
	$\frac{a}{a-1}$	$a^{0.4}$	3b	3a	d x e
4	1.333	1.742	4.95	63.5	314
3.0	1.500	1.552	5.58	47.3	264
3.3	1.434	1.612	5.32	52.5	279.2
3.22	1.450	1.597	5.38	51.2	275.2
3.07	1.483	1.566	5.515	48.5	265.9
3.17	1.460	1.5865	5.415	50.3	272.0
3.13	1.470	1.578	5.450	49.6	270.0
3.155	1.4645	1.5838	5.432	49.93	271.2
3.156	1.4640	1.5840	5.431	50.08	271.6

From table, W_k bounce = 2 = 271.3 BTU/lb air del.

r_{cb} = 3.155

$M_{bounce\ air}$ = 5.432 lbs/lb air del.

4. Clearance: $S_{bcl} = \frac{s_s}{r_{cb}-1} = \frac{I.B.2}{3a - 1} =$

= 8.13 ins.

5. Coordinat... ... the indicator card and the force-stroke diagram

a	b	c	d
Piston Position s	S^n	P #/in^{2}abs.	F 10^3#/lb air del.
ins. from Outer Position	a^{A3}	$P \frac{b_1}{b}$	10^{-3} x B.2 x c
25.63	94.0	26.46	67.7
20.63	69.1	36.05	92.3

Coordinates of the indicator card and the force-stroke diagram, (Cont.)

a	b	c	d
Piston Position s	s^n	P $\#/in^2$ abs.	F $10^3\#$/lb air del.
ins. from Outer Position	a A3	P $\frac{b_1}{b}$	10^{-3} x B.2 x c
15.63	46.9	53.00	135.7
13.63	38.8	64.10	164.1
10.63	27.4	90.80	232.6
8.13	18.8	132.30	338.2
22.63	78.8	31.54	80.9

IV. DYNAMICAL ANALYSIS

A. Basic idealizations

.12 77.85 ←

1. Character of friction force: const. magnitude, reversing sign at the end of each stroke

$$F_{fr} = \frac{W_{k\,fr/cycle}}{S_c} = \frac{\text{II.A.3} \quad \text{I.D.20}}{2 \times \text{I.B.2}} \times 12 \times 778$$

17.5

$$= 2.48 \qquad 10^3 \#/\text{stroke}$$

2. Resultant force for the first 1/2 ins. of stroke movement (For both outstroke and instroke):

$$\overline{F}_{R_{o-\frac{1}{2}"}} = \frac{2\left(F_{R_o} - F_{R\frac{1}{2}}\right)}{\left[\cos^{-1} \frac{F_{R\frac{1}{2}}}{F_{R_o}}\right]^2}$$

B. Independent variables

1. Piston length: $l = 4.08$ ft.
2. Engine piston length: $l_e = 3.168$ ft.
3. Mass density of piston: solid steel $\rho = 485$ lbs/ft^3
4. Piston solidity $\nu = 24.5$ %

C. Basic relations

1. $F_{R\ out} = F_{eng} + F_{compr} - F_{bounce} - F_{fr}$
2. $F_{R\ in} = F_{bounce} - F_{compr} - F_{eng} - F_{fr}$
3. Newton's Second Law in the form:

$$\frac{1}{2}\frac{M}{g} V_x^2 = \int_0^x F(x)\,dx = \phi(x)$$

4. $\dfrac{\tau}{\sqrt{\dfrac{M}{2g}}} = \displaystyle\int_0^x \sqrt{\frac{2g}{M}} \times \frac{dx}{V_x} = \int_0^x \frac{dx}{\sqrt{\phi(x)}}$ where $\phi(x)$ is defined above

D. Dependent variables

1. Resultant force vs. piston position for out-stroke.

Plot $F_{R\ out}$ vs. piston position, curve piston force vs. stroke[a]

a	b	c	d	e	f
Piston Position ins. from I.T.C.	F_{eng} exp. str. 10^3#	F_{compr} exp. str. 10^3#	F_{bounce} cyl. compr. str. 10^3#	F_{fr} 10^3#	$F_{R\ out}$ 10^3#
s	from Figure	from Figure	from Figure	A.1	b+c-d-e
0	594	162	67.5	2.5	66.6 666.0
0.5	431	126	71.0	2.5	483.5
1.0	343	103	72.0	2.5	371.5
1.5	277	86	74.0	2.5	286.5
2.0	231	73	77.0	2.5	224.5
2.5	199	62	79.0	2.5	179.5
3.0	176	57	81.0	2.5	149.5
3.5	157	49	82.0	2.5	121.5
4.0	143	44	86.0	2.5	98.5
4.5	131	39	88.0	2.5	79.5
5.0	121	36	91.0	2.5	63.5
5.5	112	32	94.0	2.5	47.5
6	104	30	98.0	2.5	33.5
6.5	96	30	103.0	2.5	22.5
7	90	30	107.0	2.5	20.5
7.5	85	30	111.0	2.5	1.5
8	80	30	116.0	2.5	- 8.5
8.5	76	30	121.0	2.5	-17.5

[a] Appendix A

D. Dependent variables (Cont.)

a	b	c	d	e	f
Piston Position ins. from I.T.C.	F_{eng} exp.str. 10^3#	F_{compr} exp.str. 10^3#	F_{bounce} cyl.compr. str. 10^3#	F_{fr} 10^3#	$F_{R\ out}$ 10^3#
s	from Figure	from Figure	from Figure	A.1	b+c-d-e
9.5	71	30	131.0	2.5	-32.5
9.7	70	30	133.0	2.5	-35.5/-80.5
10	25	30	137.0	2.5	-84.5
10.5	25	30	144.0	2.5	-91.5
11.0	25	30	150.0	2.5	-97.5
11.5	25	30	158.0	2.5	-105.5
12	25	30	165.0	2.5	-112.5
12.5	25	30	173.0	2.5	-120.5
13	25	30	184.0	2.5	-131.5
13.5	25	30	195.0	2.5	-142.5
14	25	30	207.0	2.5	-154.5
14.5	25	30	220.0	2.5	-167.5
15	25	30	234.0	2.5	-181.5
15.5	25	30	253.0	2.5	-200.5
16	25	30	269.0	2.5	-216.5
16.5	25	30	291.0	2.5	-238.5
17	25	30	315.0	2.5	-262.5
17.5	25	30	339.0	2.5	-286.5

2. Resultant force vs. piston position for in-stroke.

Plot $F_{R\ in}$ vs. piston position: curve Piston Force vs. Stroke[a]

a	b	c	d	e	f
Piston position ins. from I.T.C.	F_{bounce} 10^3#	$F_{eng.}$ compr. str. 10^3#	F_{compr} compr. str. 10^3#	F_{fr} 10^3#	$F_{R\ in}$ 10^3
	from Fig.	from Fig.	from Fig.	A.1	b-c-d-e
0	339	25	30	2.5	281.5
0.5	315	25	32	2.5	255.5
1.0	291	25	33	2.5	230.5
1.5	269	25	34	2.5	207.5
2.0	253	25	34	2.5	191.5
2.5	234	25	35	2.5	171.5
3	220	25	37	2.5	155.5
3.5	207	25	38	2.5	141.5
4	195	25	40	2.5	127.5
4.5	184	25	42	2.5	114.5
5	173	25	43	2.5	102.5
5.5	165	25	45	2.5	92.5
6	158	25	48	2.5	82.5
6.5	150	25	50	2.5	72.5
7	144	25	53	2.5	63.5
7.5	137	25	56	2.5	53.5
7.8	130	25	59	2.5	43.5

[a]Appendix A

D. Dependent variables (Part 2, Cont.)

a	b	c	d	e	f
Piston Position ins. from I.T.C.	F_{bounce} 10^3#	F_{eng} compr. str. 10^3#	$F_{compr.}$ compr. str. 10^3#	F_{fr} 10^3#	F_R in 10^3
	from Fig.	from Fig.	from Fig.	A.1	b-c-d-e
8	131	25	60	2.5	43.5
8.5	125	26	63	2.5	33.5
9	121	27	67	2.5	24.5
9.5	116	30	72	2.5	11.5
10	111	32	77	2.5	-0.5
10.5	107	34	83	2.5	-12.5
11	103	38	88	2.5	-25.5
11.5	98	42	96	2.5	-42.5
12	94	46	105	2.5	-59.5
12.5	91	51	114	2.5	-76.5
13	88	57	126	2.5	-97.5
13.5	87	64	139	2.5	-119.5
14	82	73	154	2.5	-147.5
14.2	81.5	77	162	2.5	-160.0
14.5	81.0	84	162	2.5	-167.5
15	79.0	99	162	2.5	-184.5
15.5	77.0	119	162	2.5	-206.5
16	74.0	147	162	2.5	-237.5
16.5	72.0	187	162	2.5	-279.5
17	71.0	266	162	2.5	-359.5
17.5	67.5	404	162	2.5	-501.0

3. Velocity and time vs. piston position for out-stroke

a	b	c	d	e	f	g	h
Piston Position ins. from I.T.C.	F_r out 10^3#	$\bar{F}_{R_{OUT}} \times \Delta X$ $(\Delta X = \frac{1}{24}')$ 10^3 Ft #	$\frac{M}{2g} V_x^2 = \int^x F_R d_x = \phi(x)$ 10^3 Ft #	$\sqrt{\frac{M}{2g}} V_x = \sqrt{\phi(x)}$ $\sqrt{Ft.\#}$	$\frac{1}{\sqrt{\phi(x)}}$ $\frac{10^{-3}}{\sqrt{Ft.\#}}$	$\left(\frac{1}{\sqrt{\phi(x)}}\right)\Delta X$ $10^{-3}\sqrt{\frac{Ft.}{\#}}$	$T/\sqrt{\frac{M}{2g}}$ $10^{-3}\sqrt{\frac{Ft}{\#}}$
	1f	$\frac{b_n + b_{n-1}}{48}$	$c_n + d_{n-1}$	$\sqrt{d \times 10^3}$	$10^3/e$	$\frac{f_r + f_{r-1}}{48}$	$g_{n-1} + h_n$
0	666	-	0	0	∞	-	0
.5	483.5	23.92	23.92	155	6.46	.484	.484
1.0	371.5	17.84	41.76	205	4.88	0.236	.720
1.5	286.5	13.74	55.50	236	4.24	.191	.911
2.0	224.5	10.66	66.16	258	3.88	.169	1.080
2.5	179.5	8.43	74.59	273	3.67	.158	1.238
3.0	149.5	6.86	81.45	286	3.50	.150	1.388
3.5	121.5	5.65	87.10	296	3.38	.144	1.532
4.0	98.5	4.58	91.68	303	3.30	.140	1.672
4.5	79.5	3.71	95.39	309	3.24	.136	1.808
5.0	63.5	2.98	98.37	314	3.19	.134	1.942
5.5	47.5	2.31	100.68	317	3.16	.133	2.075
6.0	33.5	1.69	102.37	321	3.12	.131	2.206
6.5	22.5	1.17	103.54	322	3.11	.130	2.336
7.0	20.5	.90	104.44	324	3.09	.129	2.465
7.5	1.5	0.46	104.90	324	3.09	.129	2.594
8.0	-8.5	-0.15	104.75	324	3.09	.129	2.723
8.5	-17.5	-.54	104.21	323	3.10	.129	2.852
9.0	-24.5	-0.88	103.33	322	3.11	.130	2.982
9.5	-32.5	-1.19	102.14	320	3.13	.130	3.112
10.0	-84.5	-2.44	99.70	316	3.17	.131	3.243
10.5	-91.5	-3.67	96.03	311	3.22	.133	3.376
11.0	-97.5	-3.94	92.09	304	3.29	.136	3.512
11.5	-105.5	-4.23	87.86	295	3.39	.140	3.652

a	b	c	d	e	f	g	h
Piston Position ins. from I.T.C.	F_r out 10^3 #	$\bar{F}_{R\,out} \times \Delta X$ $(\Delta X = \frac{1}{24}')$ 10^3 ft #	$\frac{M}{2g} V_x^2 = \int_0^x F_r\, d_x = \phi(x)$ 10^3 ft. #	$\sqrt{\frac{M}{2g}}\; V_x = \sqrt{\phi(x)}$ $\sqrt{\text{ft. \#}}$	$\frac{1}{\sqrt{\phi(x)}}$ $\frac{10^{-3}}{\sqrt{\text{ft. \#}}}$	$\overline{\frac{1}{\sqrt{\phi(x)}}}\,\Delta X$ $10^{-3}\sqrt{\frac{\text{ft}}{\#}}$	$\tau / \sqrt{\frac{M}{2g}}$ $10^{-3}\sqrt{\frac{\text{ft}}{\#}}$
	1f	$\frac{b_n + b_{n-1}}{48}$	$c_n + d_{n-1}$	$\sqrt{d \times 10^3}$	$10^3 / e$	$\frac{f_n + f_{n-1}}{48}$	$g_{n-1} + h_n$
12.0	112.5	-4.54	83.32	289	3.46	.143	3.795
12.5	-120.5	-4.86	78.46	281	3.56	.146	3.941
13.0	-131.5	-5.25	73.21	271	3.69	.151	4.092
13.5	-142.5	-5.71	67.50	260	3.85	.157	4.249
14.0	-154.5	-6.18	61.32	248	4.03	.165	4.414
14.5	-167.5	-6.71	54.61	234	4.27	.173	4.587
15.0	-181.5	-7.27	47.34	218	4.58	.185	4.772
15.5	-200.5	-7.96	39.38	199	5.03	0.201	4.973
16.0	-216.5	-8.68	30.70	175	5.71	0.224	5.197
16.5	-238.5	-9.48	21.22	146	6.85	0.262	5.459
17.0	-262.5	-10.45	10.77	104	9.62	0.343	5.802
17.5	-286.5	-11.46	-0.69	0	∞	0.823	6.625

4. Velocity and time vs. piston position for in-stroke

a	b	c	d	e	f	g	h
Piston Position inches from O.T.C.	F_R in 10^3 #	$\bar{F}_{R_{iii}} \times \Delta X$ $(\Delta x = \frac{1}{24}$ ft) 10^3 ft #	$\frac{M}{2g} V_x^2 = \int_0^x F_R dx - \phi(x)$ 10^3 ft #	$\sqrt{\frac{M}{2g}} V_x = \sqrt{\phi(x)}$ $\sqrt{\text{ft}}$ #	$\frac{1}{\sqrt{\phi(x)}}$ $\frac{10^{-3}}{\sqrt{\text{ft } \#}}$	$\left(\frac{1}{\sqrt{\phi(x)}}\right)\Delta X$ $10^{-3}\sqrt{\frac{\text{ft}}{\#}}$	$\tau / \sqrt{\frac{M}{2g}}$ $10^{-3}\sqrt{\frac{\text{ft}}{\#}}$
	2f	$\frac{b_n + b_{n-1}}{48}$	$c_n + d_{n-1}$	$\sqrt{d} \times 10^3$	$10^3 / e$	$\frac{f_n + f_{n-1}}{48}$	$g_{n-1} + h_n$
0	281.5	-	0	0	∞	-	0
.5	255.5	11.20	11.2	106	9.44	.778	.778
1.0	230.5	10.10	21.30	146	6.85	.339	1.117
1.5	207.5	9.15	30.45	174	5.75	.262	1.379
2.0	191.5	8.33	38.79	197	5.08	.226	1.605
2.5	171.5	7.57	46.36	216	4.63	.203	1.808
3.0	155.5	6.83	53.19	231	4.33	.187	1.995
3.5	141.5	6.17	59.36	244	4.10	.175	2.170
4.0	122.5	5.60	64.96	255	3.92	.167	2.337
4.5	114.5	5.04	70.00	265	3.77	.160	2.537
5.0	102.5	4.52	74.52	273	3.68	.156	2.693
5.5	92.5	4.06	78.58	281	3.56	.151	2.844
6.0	82.5	3.64	82.22	287	2.48	.126	2.970
6.5	72.5	3.23	85.45	292	3.42	.123	3.093
7.0	63.5	2.83	88.28	297	3.37	.142	3.235
7.5	53.5	2.44	70.72	301	3.33	.140	3.375
8.0	43.5	2.03	92.75	304	3.29	.138	3.513
8.5	33.5	1.61	94.36	307	3.26	.137	3.650
9.0	24.5	1.21	95.57	309	3.24	.136	3.786
9.5	11.5	.75	96.32	311	3.22	.135	3.921
10.0	-0.5	.23	96.55	311	3.22	.134	4.055
10.5	-12.5	-.27	96.78	311	3.22	.134	4.189

a	b	c	d	e	f	g	h
Piston Position inches from O.T.C.	F_R in 10^3 #	$\bar{F}_{R_N} \times \Delta x$ $(\Delta x = \frac{1}{24}\,ft)$ 10^3 ft #	$\frac{M}{2g} V_x^2 = \int_0^x F_R\,dx = \phi(x)$ 10^3 ft #	$\sqrt{\frac{M}{2g}}\, V_x = \sqrt{\phi(x)}$ $\sqrt{ft\ \#}$	$\frac{1}{\sqrt{\phi(x)}}\,\Delta x$ $\frac{10^{-3}}{\sqrt{ft\ \#}}$	$\left(\frac{1}{\sqrt{\phi(x)}}\right)\Delta x$ $10^{-3}\sqrt{\frac{ft}{\#}}$	$t \Big/ \sqrt{\frac{M}{2g}}$ $10^{-3}\sqrt{\frac{ft}{\#}}$
	$2f$	$\frac{b_N + b_{N-1}}{48}$	$c_N + d_{N-1}$	$\sqrt{d \times 10^3}$	$10^3 / e$	$\frac{f_N + f_{N-1}}{48}$	$g_{N-1} + h_N$
11.0	-25.5	-.79	95.49	309	3.24	.135	4.424
11.5	-42.5	-1.42	94.07	307	3.26	.136	4.560
12.0	-59.5	-2.13	91.94	303	3.30	.137	4.697
12.5	-76.5	-2.84	89.10	299	3.35	.139	4.836
13.0	-97.5	-3.63	85.47	293	3.42	.141	4.977
13.5	-119.5	-4.53	80.94	285	3.51	.145	5.122
14.0	-147.5	-5.57	75.37	275	3.64	.149	5.271
14.5	-167.5	-6.56	68.71	262	3.82	.156	5.427
15.0	-184.5	-7.35	61.36	248	4.03	.164	5.591
15.5	-206.5	-8.16	53.20	231	4.33	.174	6.765
16.0	-237.5	-9.27	43.93	210	4.76	.190	6.955
16.5	-279.5	-10.80	33.13	182	5.50	.214	7.169
17.0	-359.5	-13.33	19.80	141	7.10	.263	7.432
17.5	-501	-17.97	1.83	0	∞	.593	8.025

4a. Initial and final forces, column e, item D4

(A) $$_oF_{a\frac{1}{2}} = \frac{2(F_{R_o} - F_{R1})}{(\cos^{-1}\frac{F_{R1}}{F_{R_o}})^2} = 286000 \quad \#$$

$$\frac{_o\tau_{\frac{1}{2}}}{\sqrt{\frac{M}{2g}}} = \frac{2\sqrt{\frac{M}{2g}}}{_of_{a_{\frac{1}{2}}}} \quad V_{\frac{1}{2}} = 0.750 \times 10^{-3} \sqrt{\frac{ft}{\#}}$$

(B) $$\frac{_{17}\tau_{17\frac{1}{2}}}{\sqrt{\frac{M}{2g}}} = \frac{2\sqrt{\frac{M}{2g}} \quad V_{\frac{1}{2}}}{f_{a\ 17\frac{1}{2}}} = 0.701 \times 10^{-3}\sqrt{\frac{ft}{\#}}$$

5. Period of 1 cycle: for 1 lb. of air del./cyl.cycle

$$\frac{\tau_o}{\sqrt{\frac{M}{2g}}} = 3.h_{max} + 4.h_{max}$$

$$= 14.65 \qquad 10^{-3}\sqrt{\frac{ft}{\#}}$$

6. Frequency for 1 lb. of air del./cyl.cycle - Piston Mass Relationship

$$fr\sqrt{M} = \sqrt{\frac{64.4}{5}} = 548 \qquad \frac{\sqrt{lb}}{sec}$$

7. Piston weight

$$W_t = \rho\nu(A_{bounce}\ \ell_{bounce} + A_{eng}\ \ell_{eng})$$

$$W_t = \frac{B4\ B3}{100}\left[\frac{III\ B2}{144}(B1 - B2) + IID\ 15\ B2\right]$$

$$= 2880 \qquad lb/lb \text{ air del.}$$

8. Frequency for 1 lb of air del./cyl.cycle

548

$$fr = \frac{6 \times 60}{\sqrt{7}} = 613 \qquad \sim/min.$$

← 2880

9. Air delivery rate: for 1 lb. of air del./cyl.cycle

$$W_{air} = 2 \times fr = \frac{2 \times 8}{60} = 20.44 \qquad lbs/sec.$$

10. Piston max. acceleration:

$$A_{max} = \frac{F_{R_{max}}}{M} = \frac{2f\ max \times 32.2}{7} = 5370 \qquad fr/sec^2$$

V. OVERALL GAS GENERATOR PERFORMANCE (for 1 lb of air del./cyl. x compr. cycle)

A. Basic Idealizations

1. Equation of state of exhaust gases:

   Perfect Gas with const. specific heat capacities.

2. Pressure drop between cylinder "blow out" and turbine inlet: 5%

3. Eng. exhaust gas temp. equal to the average temperature of the "blow down" period.

4. Generator gas delivery temperature: weight average between eng. exhaust gas temp. and scavenge gas temp.

5. Turbine isentropic efficiency: 85%

B. Independent variables

1. Net power output: 1138 hp.

C. Basic relations

1. $T_{eng.\ gas} = \frac{T_4}{K}\left[1 + (K-1)\frac{P_5}{P_4'}\right]$

2. $f_r = \text{const.} = f_{r\,1\ \text{lb air del./cyl x cycle}}$

3. $M \propto D^2 \propto w_{air\ del/cyl.} \propto \text{shp}$

D. Dependent variables

1. Gas delivery rate: (for 1 lb of air del. per cylinder per cycle)

$$W_G = \left[1 + \frac{W_{air}}{f}\right] = \text{IV D.9}\left[1 + \text{II D.14} \times \frac{1}{\text{II B.1}}\right]$$

$= 20.80$ lbs/sec.

2. Gas Generator discharge pressure:

$$P_G = \left(1 - \frac{\text{A2}}{100}\right) \times \text{II D.1} = 64.5 \quad \text{p.s.i.a.}$$

3. Temperature at state 4'

$$T_{4'} = \frac{m}{R} \times P_{4'} \times v_4 = \frac{144}{\text{III C1}} \times \text{II.D.19.f.2} \times \text{II.D.3}$$

$$= 2312 \qquad {}^{\circ}R$$

4. Ratio of min. to max, pressure of "blow-down":

$$\frac{P_5}{P_{4'}} = \frac{\text{II.D.1}}{\text{II.D.19.f.2}} = 0.361$$

5. Temperature at state 5(s)

$$T_{5(s)} = T_{4'} \times \left(\frac{P_5}{P_{4'}}\right)^{\frac{k-1}{k}} = 3 \times 4 = 903 \qquad \text{deg. R}$$

6. Temperature of engine exhaust gas

$$T_{eng.\ gas} = \frac{T_{4'}}{K}\left[1 + (k-1)\frac{P_s}{P_{4'}}\right] = \frac{3}{\text{III C1}}\left[1 + \text{III C1}\ 4\right]$$

$$= 1968 \qquad \text{deg. R}$$

7. Generator gas delivery temperature

$$T_G = w_{eng} \times T_{eng.\ gas} + (1-w_{eng}) \times T_1$$

$$= \text{II.D.14} \times 6 + \left[1 - \text{II.D14}\right] \times \text{II.D.2}$$

$$= 1416 \qquad {}^{\circ}R$$

8. Available pressure ratio for turbine expansion:

$$P_t = \frac{2}{14.7} = 4.38$$

9. Enthalpy and relative pressure function of generator air at delivery:

From G.T., for $T_G = 7$ $h_G = 347.1$ Btu/lb

$$P_r = 44.78$$

10. Relative pressure function at exhaust:

$$P_{r\ exhaust} = \frac{9}{8} = 10.21$$

11. Isentropic temperature and enthalpy of generator air at exhaust pressure:

From G.T. for P_r = (10): $T_{exh\,(S)} = 950$ deg. R.;

$h_{exh\,(S)} = 228.6$ Btu/lb

12. Isentropic enthalpy drop:

$h_{(S)} = (9) - (11) = 118.5$ Btu/lb

13. Isentropic power:

$P_s = (1) \times (12) \times \frac{3600}{2545} = 3482$ HP

14. Net output for 1 lb. of air del. per cylinder per cycle:

$P_n = \frac{(13)(A5)}{100} = 2960$ HP

15. Fuel rate for 1 lb. of air del. per cylinder per cycle:

$$\frac{w_{eng} \times w_{air\ del} \times 3600}{f} = \frac{(II.D.14) \times (IV.D.9) \times 3600}{(II.B.1)}$$

$= 1186$ lb fuel/hr

16. Specific fuel rate: $\frac{(15)}{(14)} = 0.401$ lb fuel/HPhr

17. Thermal efficiency (LHV):

$$\frac{2545}{(16) \times (II\ B4)} = 0.350$$

18. Air delivered per cylinder per cycle for actual hp net output:

$w_{air} = \frac{(B1)}{(14)} = 0.385$ lb air/cyl. cycl.

19. Fuel rate for actual hp net output: $(B1) \times (16) = 456$ lb fuel/hr

20. Piston dimensions for actual hp net output:

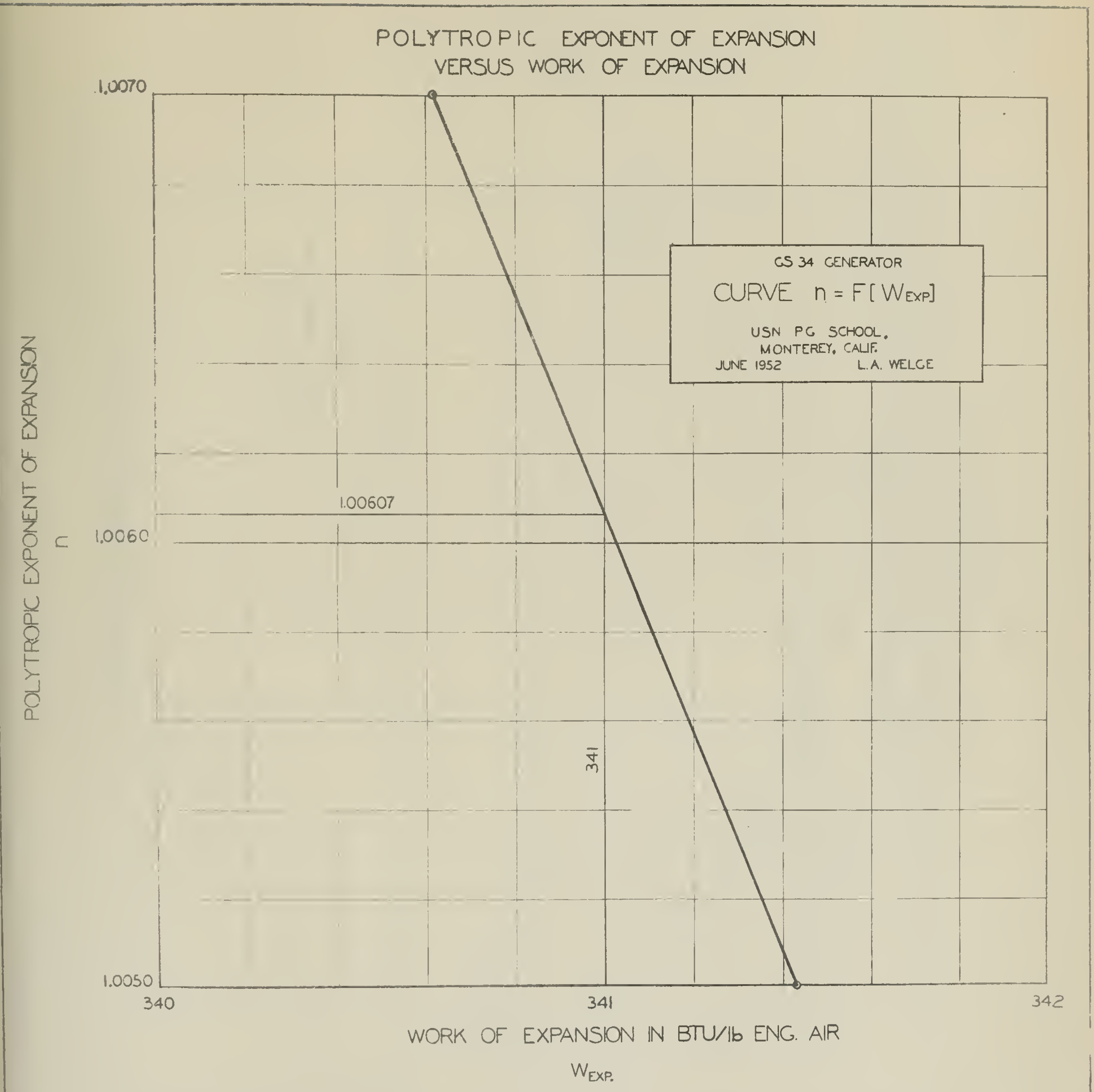
a	b		c
Piston Designation	A $\frac{ft^2}{\text{lb air del}}$		A $\frac{ft^2}{\text{Act HP}}$
			(18) x (b)
Engine	(II.D.15)	2.56	0.982
Bounce cyl.	(III.B.2)	17.75	6.82
Compressor	(I.D.11)	15.01	5.76

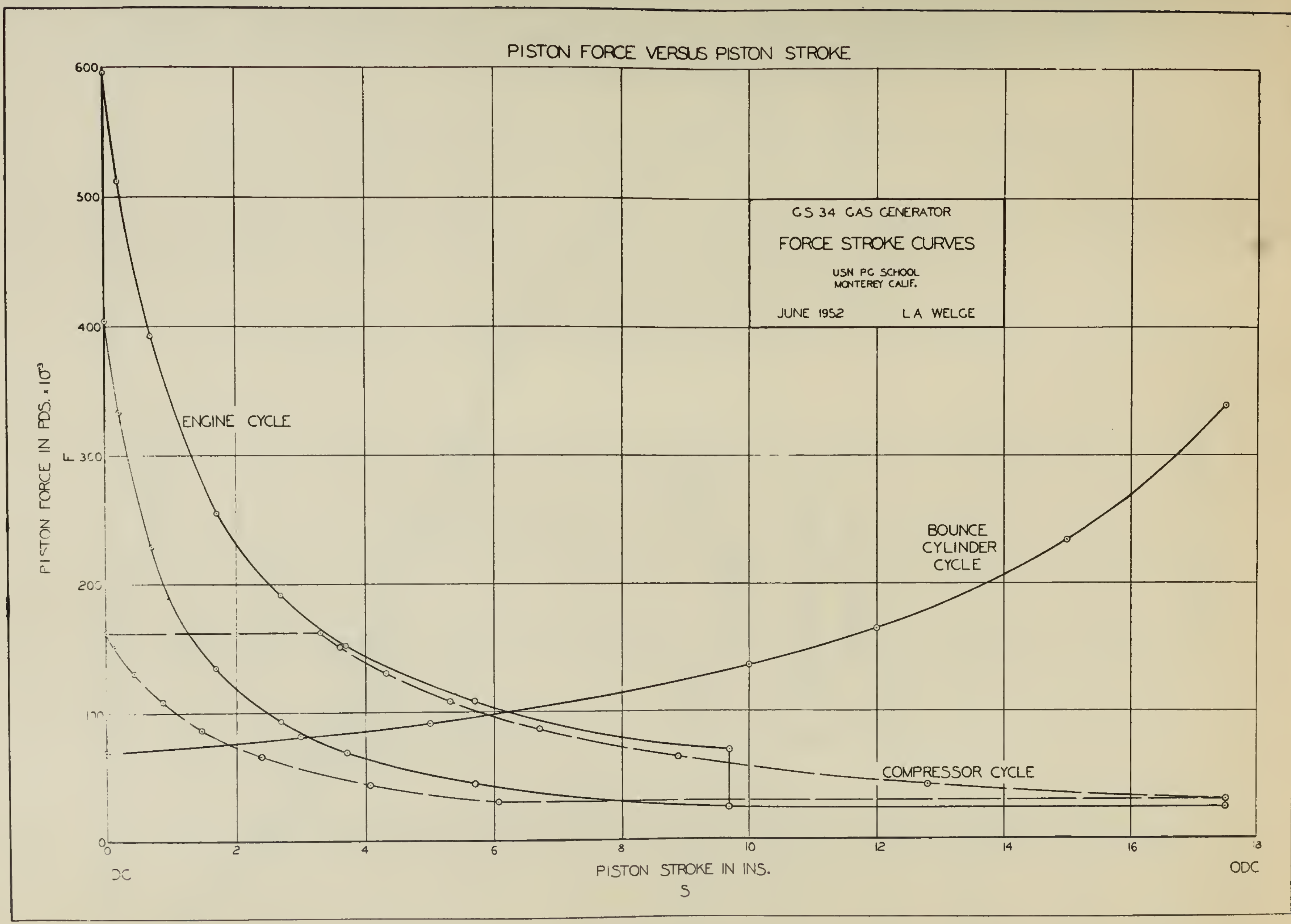
21. Pistom Mass for actual hp net output:

$$M = \boxed{IV.D.7} \times (18) = 1110 \qquad \text{lb}$$

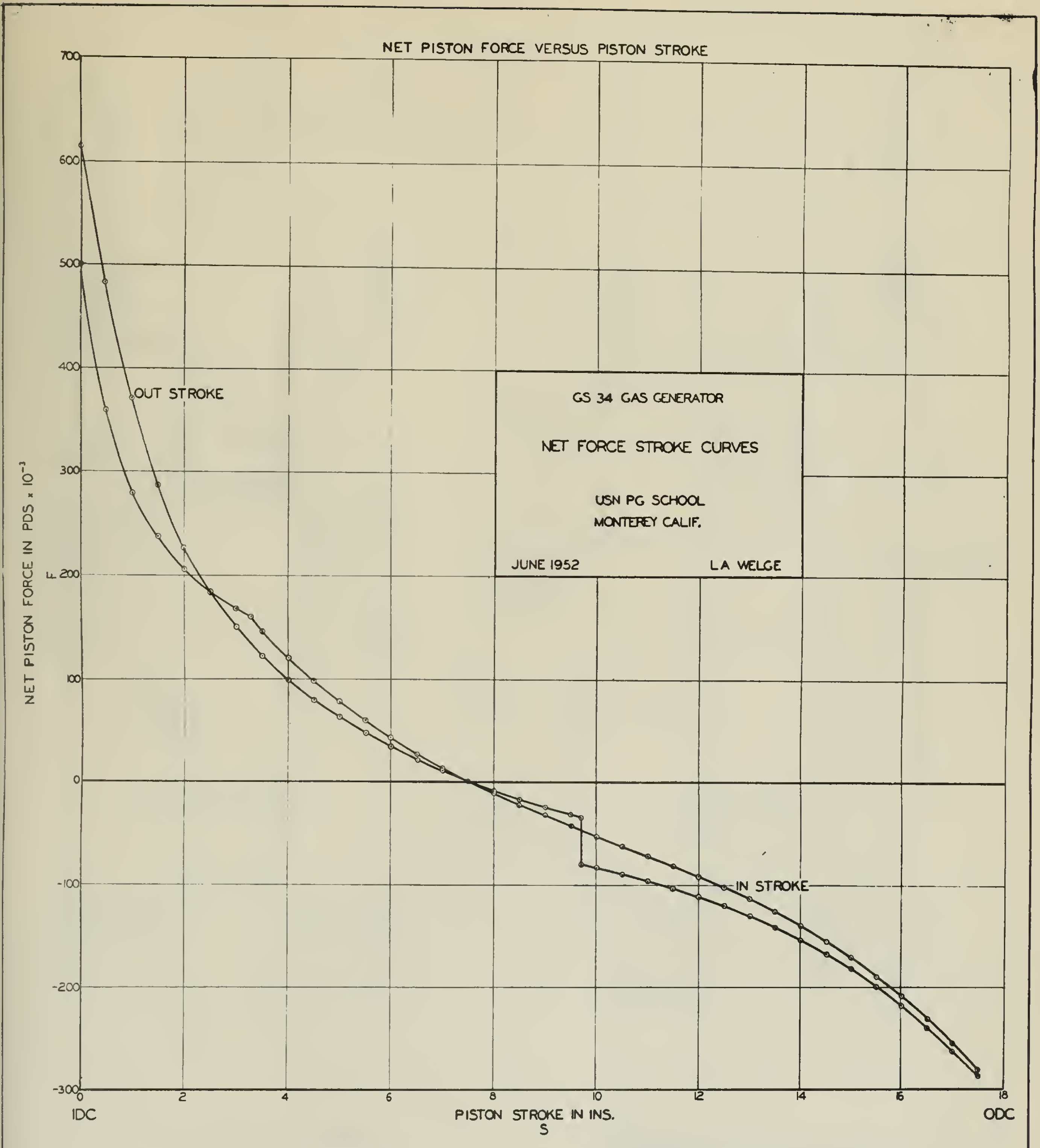
22. Gas flow rate for actual hp net output:

$$W_{gas} = (1) \times (18) \times 3600 = 28900 \qquad \text{lb/hr}$$


POLYTROPIC EXPONENT OF EXPANSION
VERSUS WORK OF EXPANSION
1.0070
1.0060
1.0050
1.00607
341
GS 34 GENERATOR
CURVE n = F[WEXP]
USN PG SCHOOL,
MONTEREY, CALIF.
JUNE 1952 L.A. WELGE
POLYTROPIC EXPONENT OF EXPANSION
n
340
341
342
WORK OF EXPANSION IN BTU/lb ENG. AIR
WEXP.



PISTON FORCE VERSUS PISTON STROKE
G.S 34 GAS GENERATOR
FORCE STROKE CURVES
USN PG SCHOOL
MONTEREY CALIF.
JUNE 1952
L A WELGE
ENGINE CYCLE
BOUNCE CYLINDER CYCLE
COMPRESSOR CYCLE
PISTON FORCE IN PDS. x 10³
F
PISTON STROKE IN INS.
S
ODC
600
500
400
300
200
100
0
2
4
6
8
10
12
14
16
18



NET PISTON FORCE VERSUS PISTON STROKE
GS 34 GAS GENERATOR
NET FORCE STROKE CURVES
USN PG SCHOOL
MONTEREY CALIF.
JUNE 1952
L A WELGE
OUT STROKE
IN STROKE
NET PISTON FORCE IN PDS x 10^{-3}
F
PISTON STROKE IN INS.
S
IDC
ODC
700
600
500
400
300
200
100
0
-100
-200
-300
0
2
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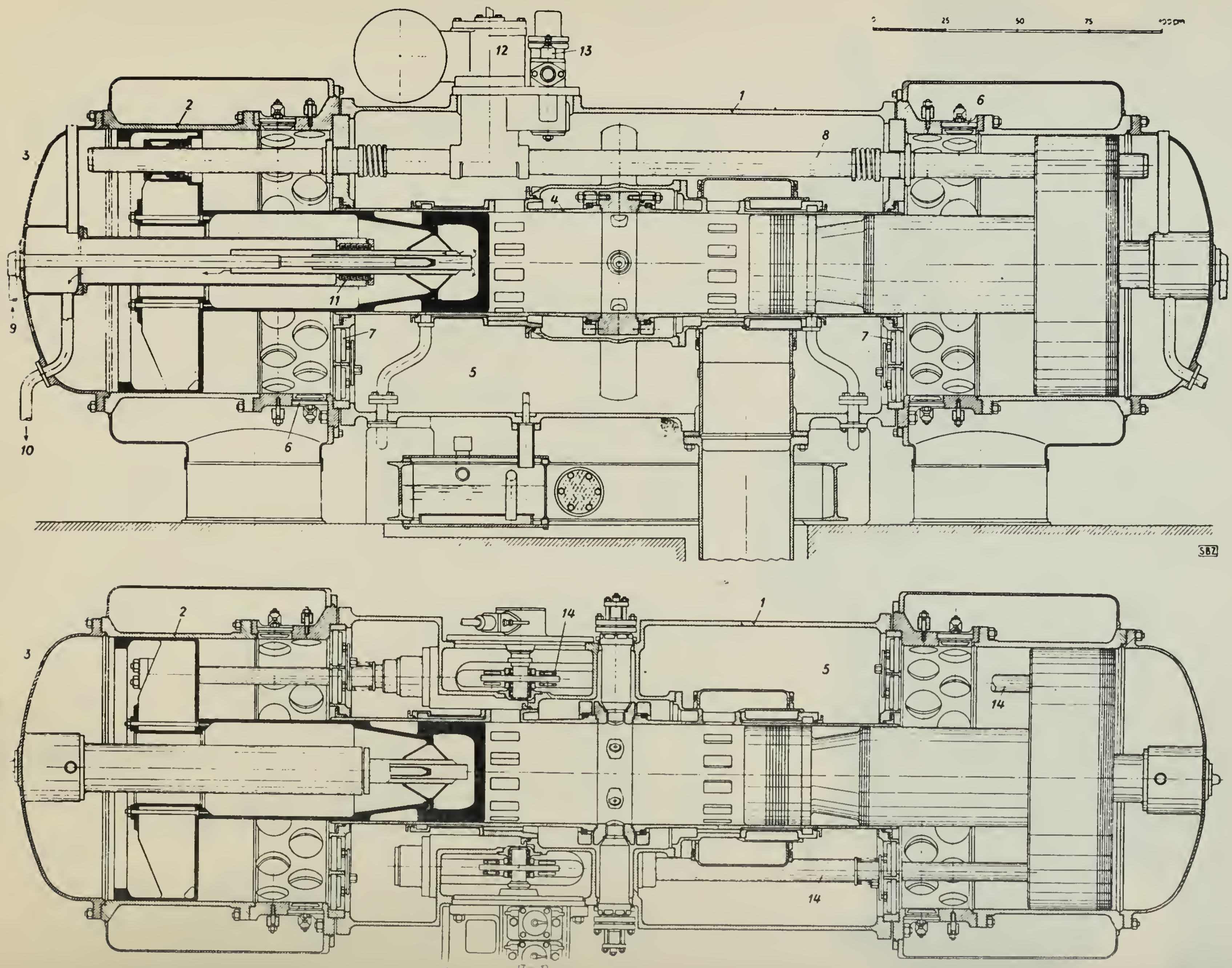
18



0
25
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100 cm
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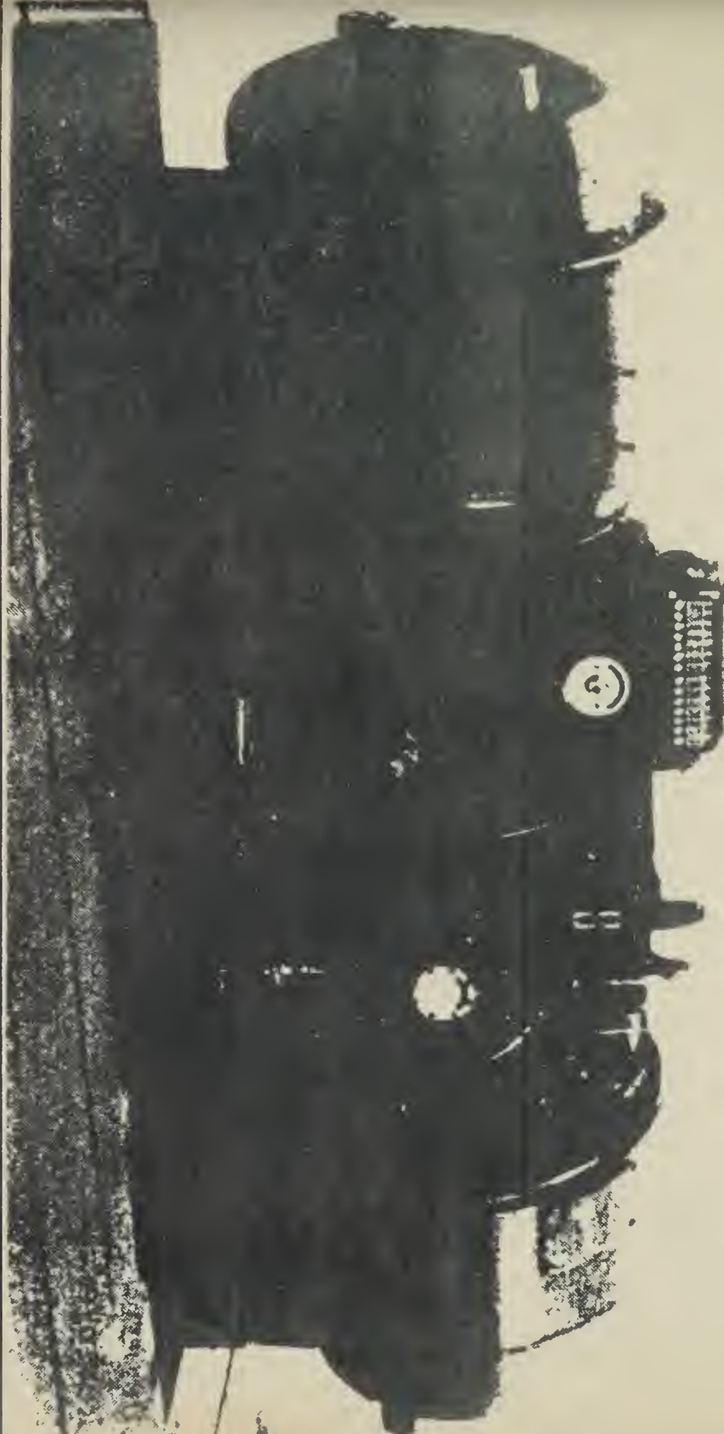
SBZ



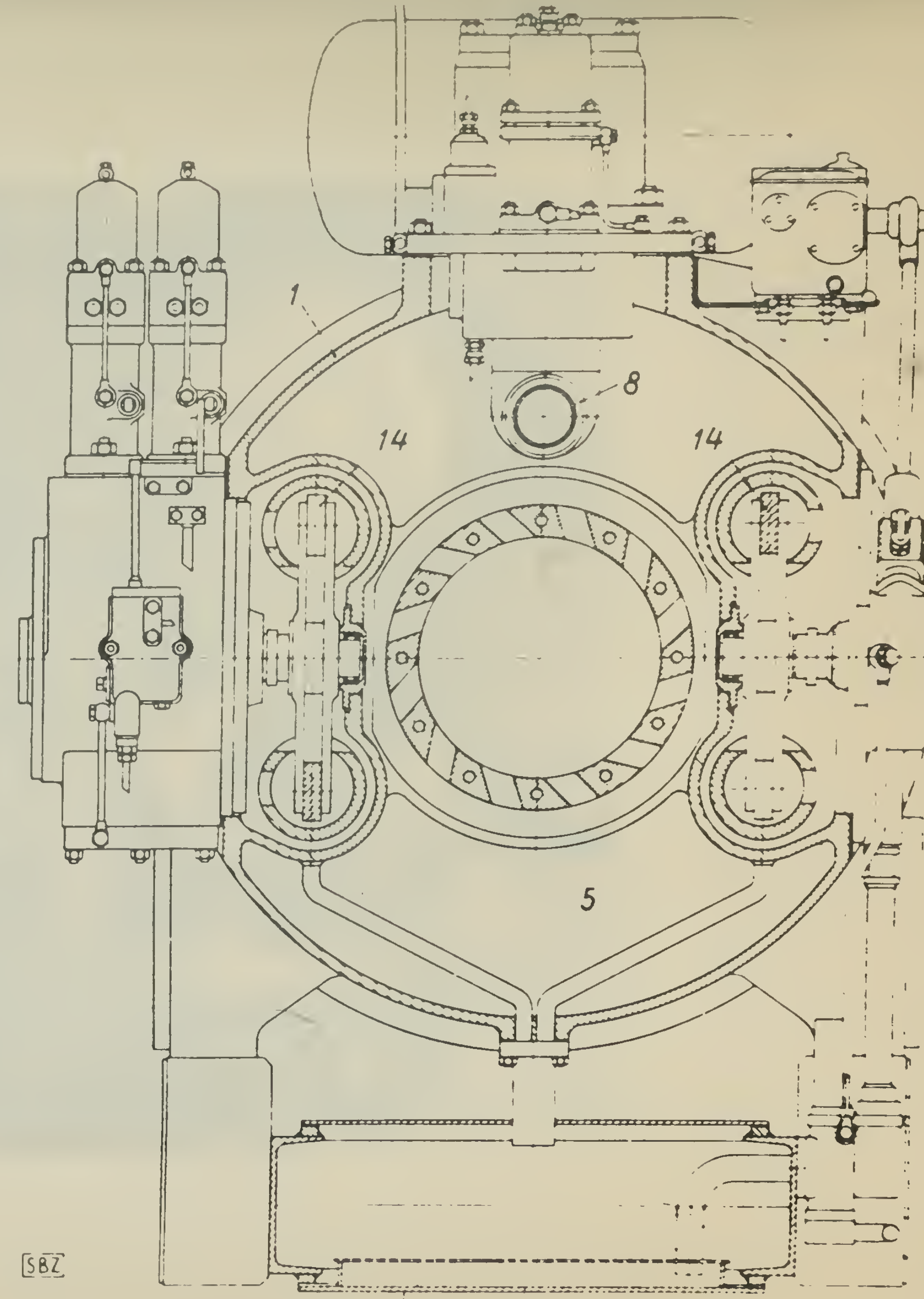
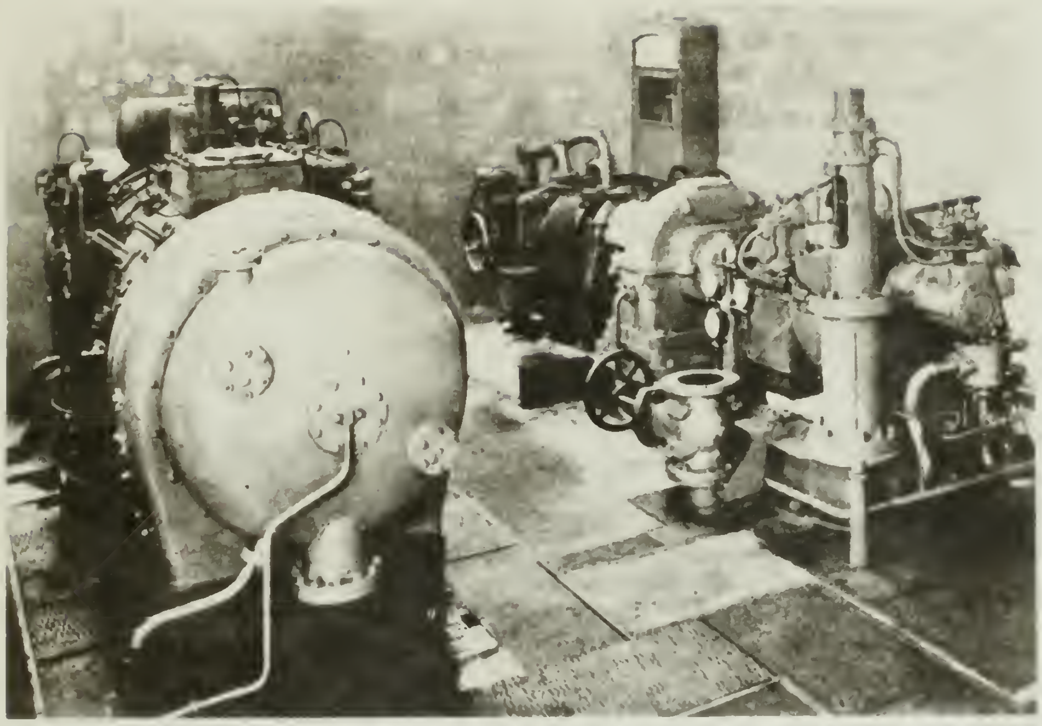


Bild 12. Querschnitt zu den Bildern 10 und 11



All machinery views courtesy A.L. London

THE FREE PISTON GAS GENERATOR

APPENDIX B

INHERENT CHARACTERISTICS OF THE MULTIPLE FREE PISTON GAS GENERATOR TURBINE PRIME MOVER

THE INHERENT CHARACTERISTICS OF THE FREE PISTON GAS GENERATOR TURBINE PRIME MOVER

The subject prime mover has many unique characteristics deserving detailed consideration. It was felt the study should be limited to consideration of inherent and proven gas generator characteristics in comparison with the characteristics of other prime movers. To these ends, a table of prime mover characteristics was prepared[a] which was amplified by the following discussion.

EFFICIENCY

The S.I.G.M.A. GS34 Model with an 85 percent efficient turbine operated at an efficiency of 34.5 percent, at more than rated load[b]. This efficiency allows the machine to be competitive with heavy Diesel engines and other prime movers in the medium power ratings.

EFFICIENCY OVER A WIDE RANGE OF POWERS

Provided two or more gas generators are incorporated, the subject system is efficient over a wide range of powers. For although an individual gas generator does not have a desirably broad range of efficient operation, by proper power combinations of the several machines the efficiency of the system may be kept at a desirable level. Paralling the gas generators is not difficult and results in little additional weight[c].

[a] Enclosed, Page 62
[b] Appendix A

[c] Eichelberg (3)

COMPARISON OF CHARACTERISTICS OF PRIME MOVERS

Prime Mover Characteristics	Subject System	Gasoline Engine	Heavy Diesel	Boiler Turbine	Simple Gas Turbine
Thermal Efficiency	35	25-30	30-35	25-28	15-20
Range of High Efficiency	Good	Poor	Good	Good	Poor
lbs. wt. per HP (2000 HP.)	Good	Excellent	Fair	Poor	Excellent
Lack of Vibration	Excellent	Poor	Poor	Excellent	Excellent
Warm Up Period	Short	Short	Moderate	Extensive	Short
Foundation Requirements	Light	Moderate	Heavy	Complex	Light
Fuel Unit Cost	Moderate	Expensive	Moderate	Cheap	Moderate
Compactness (2000 HP)	Good	Excellent	Fair	Poor	Excellent
Critical Materials	Little	Some	Some	Variable	Extensive
Initial Cost	Cheap	Cheap	Moderate	Expensive	Expensive

COMPARISON OF CHARACTERISTICS OF PRIME MOVERS (Cont.)

Prime Mover Characteristics	Subject System	Gasoline Engine	Heavy Diesel	Boiler Turbine	Simple Gas Turbine
Effort, tools, skill in manufacture	Low	Moderate	High	Highest	High
Development Possibilities	Definite	Limited	Limited	Limited	Limited
Starting Torque	Full	None	None	Full	None
Braking Torque	None	Frictional	Frictional	None	Inherent

SPECIFIC WEIGHT

It has been pointed out[a] that the S.I.G.M.A. GS34 model had a specific weight of twenty-eight pounds per horsepower. This machine was designed for stationary power applications, and no efforts were taken to lighten the machine. Thus in comparison with other prime movers of similar purpose the specific weight is very low, further, S.I.G.M.A. is now developing a truck engine to have a specific weight of five pounds per horsepower.

LACK OF VIBRATION

The subject device is vibrationless since the two main moving parts of the machine are limited to simple motions and are linked and balanced inherently. In consequence the vibration characteristics of the system will depend on the characteristics of the turbine, the drive train and the driven unit.

The lack of vibration of free piston machinery is spectacularly demonstrated when a coin on edge is balanced easily on a loosely mounted unit under full load.

WARM-UP CHARACTERISTICS

The subject machine has an unavoidably short warm-up period. The characteristically high idling fuel consumption rate results in the machine being brought to normal operating temperature rapidly. High fuel rates are required since the fuel rate determines piston stroke length, and the stroke must be long enough to expose both cylinder parts.

[a]Eichelberg (3)

FOUNDATION REQUIREMENTS

Since the gas generator transmits no system forces or moments to its foundation, develops no vibrations, is light and transfers no complex thermally developed forces, the unit may have a light and simple bed. The associated turbine need not have a foundation in common or even in proximity to the gas generators.

FUEL UNIT COST

The subject system operates on Diesel type fuels of low or usual cetane ratings. This type fuel is not hazardous and is commonly available.

COMPACTNESS

The subject prime mover is compact, encompassing about one quarter the volume of a comparable Diesel engine in the case of the S.I.G.M.A. GS34 model.

CRITICAL MATERIALS

The gas generator presently requires little critical material in its manufacture. Since the associated turbine is not subjected to high temperatures and pressures, common steels are satisfactory in its construction also. However, when machines of higher capacities and efficiencies are developed, high alloy or special steels may well be required to some intent.

INITIAL COST

The initial cost of the gas generator is low, consistent with the simplicity and ease of manufacture and the low cost cost of construction materials. The cost of the associated turbine is less modest, but with the present moderate specifications its cost is not prohibitive.

EFFORT, TOOLS AND SKILL IN MANUFACTURE

Section plans of a gas generator[a] show the relative simplicity of the unit. Machine tools required to manufacture the machine are at a minimum; common foundry facilities and techniques are sufficient. Although fuel system components require special facilities and skills for manufacture, such are available in the industrial areas.

DEVELOPMENT POSSIBILITIES

The gas generator system is in the infancy of its development yet present models are competitive with prime movers of advanced design, the products of many years of research. It may be concluded that the gas generator system will be subjected to wide application in the industry, subsequent to its indicated development.

TORQUE CHARACTERISTICS

The subject system develops a desirably high locked shaft torque. In a manner depending on the turbine characteristics, developed torque drops with increased shaft speed until no load speed is reached where net torque is zero. However, a turbine is a unidirectional low friction device and cannot produce effective braking or reversing torques. Also, a turbine is most compact and efficient when designed for and operated at low torques and high speeds. But many applications call for low speeds, high torques, and braking or reversing torques. In consequence, the subject system will not usually require clutching and transmission mechanisms but depending on the application, may require reduction gearing or a reversing turbine.

[a] Appendix A

GAS FLOW RATE

The subject system is characterized by high flow rates of air. The exhaust is at an elevated temperature. Because of the considerable excess air and high gas temperature, no smoke or carbon monoxide is normally present in the exhaust.

STARTING CHARACTERISTICS

Compression temperatures are characteristically high. Cold starting conditions are not a problem. Since one stroke of the piston assembly is required to achieve injection and the unit has little friction, a minimum of power is required of the starting system. Compressed air, electrical or spring starting systems are practical and have been used successfully.

ENGINE LIFE

The gas generator's engine life is relatively short. Rings, liners and pistons wear rapidly when exposed to high engine temperatures and pressures, and low temperatures and pressures result in low system capacity and efficiency. Thus, despite any possible ingenuity in design, it would seem reasonable to suppose that the engine components will never be characterized by long life except at the cost of possible efficiency or capacity.

However, the Pescara type machine is characterized by a basic simplicity allowing rapid and inexpensive maintenance. In an acceptance test of a 1000 horsepower machine one piston was removed in less than forty-five minutes[a] for example.

[a]Reported by S.I.G.M.A.

THE FREE PISTON GAS GENERATOR

APPENDIX C

THE INDICATED APPLICATIONS

OF THE

FREE PISTON GAS GENERATOR-TURBINE PRIME MOVER

TRANSPORTATION

AIRCRAFT

From consideration of the specific weight of present models of gas generators, it is apparant that this new prime mover cannot shortly compete with conventional engines. In addition, the allowable shape and size of an aircraft prime mover is necessarily limited, and dependability is a primary consideration. Thus extensive research of a specialized nature would be necessary before the gas generator could be considered for powering aircraft.

LARGE SHIPS

Forced draft blowers can be powered by gas generator turbine systems, and the turbine exhausts fed to the furnaces. This is to an advantage since there is considerable oxygen still available in these high temperature gases. As appropriate, part of the intake air can be taken from the working spaces to aid in ventilation and cooling.

Occasionally steam pressure is not available or is lost through casualty. An independent source of forced draft blower power greatly facilitates establishment of normal boiler service. Otherwise the procedure is slow and hazardous.

SMALL SHIPS

Vessels too small to use steam power may have the advantage of both steam and Diesel drives by using the turbine multiple gas generator system. If so powered, low and high speeds may be achieved dependably and efficiently. In contrast, Diesel drives are characterized by poor performance at low propellor speeds.

The flexibility of the turbine-multiple gas generator system is desirable. Removal, repair, or casualty to one gas generator need not prevent control of all screws, but merely limit possible power output. Because of water tight integrity and stability requirements, the flexibility possible in locating the desirably small gas generators is an advantage. The location of propellor shafts need effect only the placement of turbines and any reduction gearing.

HEAVY VEHICLES

There is promise for gas generator turbine heavy vehicle drives. The more complex the usual drive transmission system required, the more likely a gas generator turbine system will be more suitable. Vehicles that have multiple engine functions such as fire trucks, crane trucks, and the like, can gain additionally by a gas generator turbine drive. A turbine can be provided for propulsion and a turbine for each auxiliary job, both served by the same gas generators. Thus in a fire truck pumper, by simple valving, the main gas generators can provide power for both water pump and truck propulsion, eliminating the extra engines or extensive transmission and clutching systems necessary in conventional installations.

In special purpose heavy military vehicles there is promise for the gas turbine system. In a tread vehicle, each tread could be powered by its own turbine so that vehicle control would be a matter of selective gas valving rather than the conventional heavy and complex drive mechanisms. Of particular interest are the high starting torques inherent in the subject system and the low fire hazard of heavy fuels.

The gas generator system is readily prepared for amphibious or underwater service. The few outer case joints, few shaft seals, few vents, and the auto ignition feature make this modification less difficult than in conventional drives.

Experimental trucks have been built in this country powered by simple gas turbines. Despite the poor economy, this installation was tried since trip times could be decreased materially. More fuel can be carried, decreasing the number of fuel stops. This installation also results in time saving in traffic and mountainous terrain because of high accelerating torques, high braking torques, and elimination of gear shifting.

If the simple gas turbine is considered to be practical despite its poor economy, then the turbine multiple gas generator system should be ideal, since its efficiency is that of the Diesel engine drive and otherwise it has the advantages of the gas turbine drive.

In France an eighteen ton truck is being designed to use two new type S.I.G.M.A. 120 horsepower 550 pound gas generators[a]. Thus it is apparent that the advantages of the gas generator drive are exciting practical interest in the trucking industry.

AUTOMOBILES

The characteristics of the gas generator are such that an automobile could be advantageously powered by a gas generator turbine system.

[a] Reported by the manufacturer

However, the mass production cheapness, the fine performance, and the industry's investment in present engines will always result in a minimum of demand for a different automobile power plant, despite any advantages of an alternate drive.

SUBMARINES

The gas generator turbine system is suited for submarine main drives. But one critical disadvantage is the increased size of intake and exhaust ducts necessary over conventional drives.

STATIONARY APPLICATIONS

The subject system is suitable for many medium power applications throughout the industry. S.I.G.M.A. has installed gas generators for powering electrical plants of modest capacity, and the Cooper-Bessemer Company is developing gas line prĕssure booster plants incorporating gas generator drives.

In addition, many power plants have steam systems not in optimum balance for one reason or another, and the appropriate use of gas generators for powering station auxiliaries can often result in a bettering of over all effeciencies at a modest investment.

PORTABLE POWER PLANTS

The gas generator system is small, light and flexible in arrangement, and is thus suitable for portable power plants. Of interest are emergency and military applications. The turbine and driven unit may be mounted on a trailer. The gas generators may be mounted in an associated tractor alternately as a source of power for the turbine and propulsion for the trailer.

Provided other special purpose trucks associated with the same activity are powered by gas generators an extreme flexibility is possible, for idle trucks may have their gas generator discharges coupled to any power turbines by portable piping sections. Thus the effects of loss and damage of particular gas generators are minimized and the power turbine may be operated at a load exceeding the capacity of its tractor units alone.

APPENDIX D

THE FREE PISTON GAS GENERATOR

THERMODYNAMIC-DYNAMIC ANALYSIS

AND

AFFINITY RELATIONS*

*Excerps from Oppenheim and London (1)

FREE-PISTON THERMODYNAMIC-DYNAMIC ANALYSIS

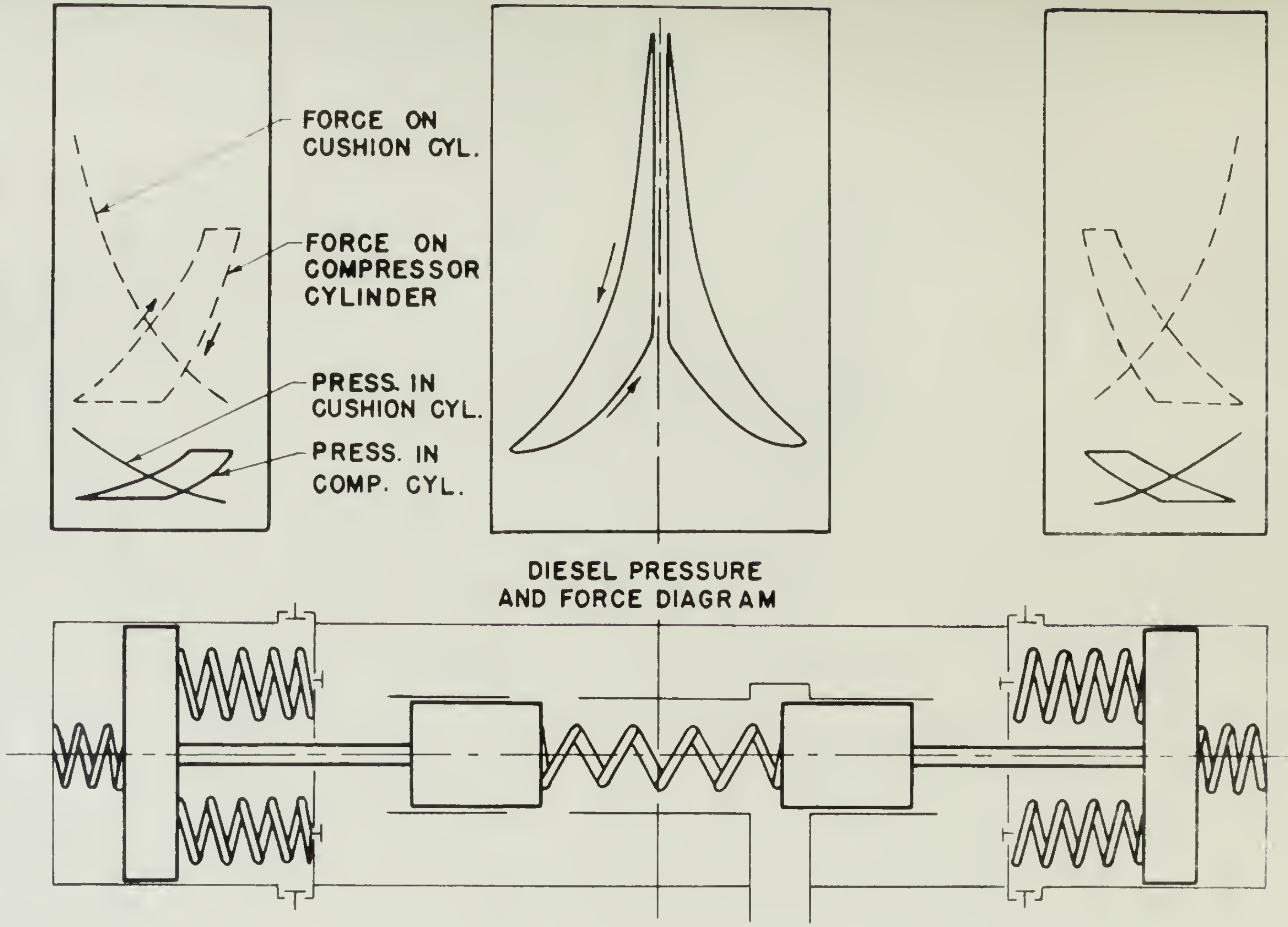
INTRODUCTION

Figure 1-I illustrates schematically the spring-mass nature of the free-piston system. The frequency is determined by the mass of a piston assembly, M, and the character of the non-linear "gas-springs". For linear springs which can be characterized by a spring constant, k, equal to the force per unit displacement, the natural frequency, as determined by elementary theory of vibrations, is

$$f_n = \frac{1}{2\pi}\sqrt{\frac{k}{M}}$$

However, because of the non-linear character of the free-piston system gas springs, no such simple equation can be used and the speed of operation must be determined from the differential equation of motion by successive graphical or numerical integrations.

Not only is the speed a dependent variable, but the stroke of the system and hence the engine compression ration and compressor cylinder clearance volumes are also variables dependent on the load conditions. These factors account for the difficulty of analysis of the free-piston system relative to the conventional crank engine. The conventional crank mechanism allows the designer to make the parameters of speed and stroke independent variables directly under his control. Further, these parameters for the engine and compressor may be selected independently. The greater number of independent variables reduces the number of dependent variables, and it is axiomatic therefore that the difficulty of analysis is substantially reduced for the crank engine-compressor system.



SPRING-MASS ANALOG FOR THE
POWER GAS GENERATOR SYSTEM

The objective of this section is to outline in detail the thermodynamic-dynamic design procedure for a free-piston system. This will be accomplished by completely defining a design problem so as to make the discussion as specific as possible. The parameters will be classified into dependent and independent variables. Idealizations necessary for the analysis will be specified. Then the basic relations defining the restraints on the system will be presented. This information provides the necessary background for the outline of the analytical procedure.

THE DESIGN PROBLEM

The general design problem can be simply stated as follows:

> It is required to determine the arrangement, dimensions and operating characteristics of a gas generator having sufficient capacity for the operation of a turbine of a given power output.

The solution of such a problem requires, respectively:

(1) Determination of the plant layout

(2) Thermodynamic-dynamic analysis

(3) Machine design analysis

The first step consists of selecting the most convenient arrangement of the engine-compressor combination out of several possibilities. The action of a free-piston engine, as it was pointed out in the introduction, depends on the storage of a part of the energy from the engine expansion stroke, which, subsequently, is utilized for engine compression in the next stroke. This action is referred to as the

bounce action, and it can be provided in a number of different ways. Excluding the possibility of utilizing a metal spring for this purpose, as presenting too great demands on the elastic properties of the metal, the bounce action can be provided by "gas springs", utilizing the expansion stroke of the compressor, as done, for example, in the Junkers free-piston compressor, or by using a special bounce cylinder. The latter solution is more convenient for the free-piston gas generator since the bounce cylinders help to maintain more stable operating conditions for the engine, result in a more flexible performance of the unit, offer additional means for synchronization of the pistons, and also can be utilized to provide a suitable phase adjustment of a number of units, should it be desired to operate two or more generators in parallel to supply gas for one turbine. All these features have been taken advantage of by Pescara and are described in Eichelberg's report[a] on the development of Pescara free-piston gas generators.

As to the arrangement of the bounce cylinder with respect to the compressor and engine cylinders, there are again several possibilities. The unit can be made symmetrical or assymmetrical, each with a variety of arrangements. The compression can be performed in one or in several stages, the bounce cylinders can be atmospheric or pressurized, etc. The basic symmetrical arrangements are discussed by Eichelberg. They include the cases of double or single acting compressors and different cylinder locations.

[a]Eichelberg (3)

The most convenient arrangement is that of a single acting compressor and bounce cylinders situated on the outside and the reasons for it, as pointed out by Eichelberg, are as follows:

(a) The unit becomes shorter than for any other case.

(b) The bounce cylinders can have a large cross-sectional area requiring, consequently, a comparatively low pressure.

(c) The bounce cylinder supplies a high return force at the same time when the compressor and the engine offer the lowest resistance, both being at the beginning of compression. The result is a high acceleration of piston return stroke and subsequently, an increase of the cycle frequency.

(d) Easier access to engin pistons than that of any other arrangement.

This arrangement has been selected also in this report for the solution of the problem primarily because of its simplicity so that the method of solution, represented here, will be as clear as possible.

The second step in the design, namely, the thermodynamic-dynamic analysis of the system, forms the main objective of this report, and it is dealt with here in detail.

The most important results of this step are numerical magnitudes for the following:

(a) Thermodynamic state (pressure and temperature) of the generated gas.

(b) Generated gas flow rate.

(c) Bores and lengths of all the cylinders.

(d) Cycle frequency.

(e) Fuel flow rate (and air flow rate).

(f) Thermal efficiency (and fuel economy).

This step of the design procedure should also include the prediction of part load characteristics for the generator and the system. However, in this report only the full load characteristics are considered.

The machine design analysis, would consist of all the usual design considerations which would determine eventually the detailed construction of the unit, its dimensions and materials, and also the details of the fuel injection and lubrication systems. Provision must also be made to synchronize the motion of the opposed piston assemblies. All these machine design considerations are outside the scope of this report. The only geometrical information needed for the dynamic analysis as independent variables are the "nominal" stroke, the piston length, and the piston average bulk density.

Only a tentative specification of the magnitude of these parameters is necessary for the second design step, since, once the solution of this step is obtained, it can be easily transformed to fit any other magnitudes of the stroke, the length, and the bulk average density of the piston assembly by the use of affinity relationships.

The design problem dealt with in this report can now be stated more specifically as follows:

Determine the full load operating characteristics and the cylinder bores of a symmetrical, one-stage, direct bounce, free-piston gas generator having sufficient capacity for the operation of a given turbine.

INDEPENDENT VARIABLES

The problem, as stated above, is still incompletely defined since the analysis depends on the magnitude of several as not yet specified independent variables and idealizations.

The independent variables for the thermodynamic-dynamic analysis of the free-piston generator-turbine system are as follows:

A. SYSTEM

(1) Net power output

(2) Stroke length common to all pistons

(3) Piston length - including length of engine piston

(4) Piston average bulk density

B. COMPRESSOR

(1) Pressure ratio

(2) Clearance

(3) Air-intake state

C. ENGINE

(1) Fuel-air ratio

(2) "Effective" stroke

(3) Compression ratio

(4) Lower heating value of fuel

D. BOUNCE CYLINDER

(1) Pressure and Temperature level (i.e., state at beginning of compression)

(2) Piston area relative to compressor area

It can be seen that only the "SYSTEM" independent variables from the above list have been specified heretofore.

The difference between the above listing and the specifications necessary for the analysis of a crank system are as follows:

(a) Stroke lengths can be selected independently for engine and for compressor.

(b) Piston length and bulk average density do not affect the analysis.

(c) Bounce cylinders are not used.

(d) Engine speed and compressor speed are independent variables and can be selected independently of each other; whereas, for the free-piston system, these parameters form a single dependent variable, the engine and the compressor operating at the same speed as determined by the dynamic, "springmass" characteristics of the system.

BASIC ASSUMPTIONS

Many idealizations must be made to accomplish the analysis. These are as follows:

(1) Nature of working substance; e.g., air and perfect gas behavior.

(2) Friction force (or friction work) as a function of stroke

(3) Details of engine cycle; e.g., its relationship to an Otto cycle

(4) Character of the change of state in flow through the scavenge chamber

(5) Character of component processes; e.g., magnitudes of the polytropic exponents.

(6) Valve pressure drop allowance

(7) Leakage allowance

(8) Degree of internal reversibility of the bounce cylinder processes

(9) Character of the engine "blow-down" process and its influence on the engine exhaust gas temperature

(10) Relationship between the gas delivery temperature, the engine exhaust gas temperature, and the scavenge gas temperature

(11) Turbine isentropic efficiency

Of the above, assumptions 2, 8, 9, 10, and 11 are not needed for the crank system. Instead of assumptions 2, 9, and 10, only estimates of the final effections of friction and of exhaust gas temperature on the energy balance are of some importance. Assumptions 8 and 11, referring to the action of bounce cylinders and of the turbine respectively, are not needed simply because neither of these is included in the crank system.

As far as the necessary estimate of the friction force and valve pressure drop is concerned, it does not affect the solution significantly and, consequently, the assumption of these quantities need not be made with great accuracy.

BASIC RELATIONS

The system is constrained by basic physical laws, the application of which to the present problem is referred to as "BASIC RELATIONS". They are as follows:

(1) Equation of state:

Here the idealization is made that the engine and compressor working substances are perfect gases, and for any particular processes the specific heats c_p and c_v are constant. Thus $PV = \frac{R}{m}T$

and
$$c_p = \frac{k}{k-1} \frac{R}{m}$$
$$c_v = \frac{1}{k-1} \frac{R}{m}$$
$$k = \frac{c_p}{c_v}$$

(2) Work balance:

$$Wk_{\substack{\text{engine}\\ \text{exp stroke}}} = Wk_{\substack{\text{bounce cyl.}\\ \text{compr stroke}}} - Wk_{\substack{\text{compressor}\\ \text{exp. stroke}}} + Wk_{\substack{\text{friction}\\ \text{per stroke}}}$$

$$Wk_{\substack{\text{bounce cyl.}\\ \text{exp. stroke}}} = Wk_{\substack{\text{engine}\\ \text{compr stroke}}} + Wk_{\substack{\text{compressor}\\ \text{compr stroke}}} + Wk_{\substack{\text{friction per}\\ \text{stroke}}}$$

$$Wk_{\substack{\text{engine}\\ \text{cycle}}} = Wk_{\substack{\text{compressor}\\ \text{cycle}}} + Wk_{\substack{\text{friction}\\ \text{per cycle}}}$$

(3) Newton's Second Law amy be stated in the form

$$F_R(x) = \frac{M}{2g} \, d(V_x)^2$$

then $$V_X^{\ 2} = \frac{2g}{M} \int_0^x F_R(x)\, dx = \frac{2g}{M} \phi(x)$$

where $\phi(x)$ is defined as $\int_0^x F_R(x)\, dx$, and resultant force $F_R(x)$ may be evaluated from

$$F_R(x)_{\text{out-stroke}} = F_{eng.} + F_{compr.} - F_{bounce} - F_{fr}$$

$$F_R(x)_{\text{in-stroke}} = F_{bounce} - F_{compr.} - F_{eng.} - F_{fr}$$

From the definition $V_x = dx/d\mathcal{T}$, it follows that

$$\mathcal{T}(x) = \int_0^x \frac{dx}{V_x}$$

Then introducing V_x as a funtion of ϕ (x) from equation (5)

$$\frac{\mathcal{T}(x)}{\sqrt{\frac{M}{2g}}} = \int_0^x \frac{dx}{\sqrt{\phi(x)}}$$

For computational convenience and accuracy, the time increment at the start of each stroke, where V_x is that of a small magnitude, was calculated from:

$$\frac{\mathcal{T}_{o-x_1}}{\sqrt{\frac{M}{2g}}} = \frac{2\sqrt{\frac{M}{2g}}\, V_{x\,1}}{Fa} = \frac{2\sqrt{\phi\,(x_1)}}{Fa}$$

where (x_1) is definnd as $(F_a)(x_1)$

and $$F_a = \frac{F_{Ro} + F_{Rx_1}}{2}$$

While the foregoing method for evaluating $\int_{o-x_1}$ was used in the calculations reported here, Appendix 2 of this section describes a more accurate procedure. This method assumes a linear $F_R(X)$ relation for the terminal stroke increments instead of a constant F_a. The recommended procedure yields predicted frequencies of the order of two percent higher than those calculated in this section.

(4) The principle of conservation of energy as applied to the exhaust blow-down was used to evaluate the average exhaust gas temperature. The result is as follows:

$$T_{eng\ gas} = \frac{T_{4'}}{k} \left[1 + (k-1) \frac{P_5}{P_{4'}} \right]$$

where 4' and 5 denote states of exhaust gases within the engine cylinder at the beginning and the end of the "blow-down" period respectively.

(5) The various criteria of similarity were used to establish affinity relationships. The application of these to the solution of the present problem is as follows:

For constant piston length, ℓ; piston bulk average density ρ; stroke, s, and shaft horsepower,

$$f_r = \text{constant}$$

$$\text{and } M \propto D^2 \propto w \left(\frac{\text{air del}}{\text{cyl. cycle}}\right) \propto \text{shp}$$

These are used to extrapolate the thermodynamic - dynamic design calculations, based on the delivery of one pound of air per cylinder cycle, to the system

dimensions required for a specified power gas delivery rate. This delivery rate in turn is determined by the net turbine power desired, the turbine isentropic efficiency, and the gas generator delivery pressure and temperature.

An overall energy balance on the power gas generator is not used directly in the analysis. It serves therefore as a useful check on the validity of the idealizations, yielding such information as the heat transfer to the engine coolant.

ANALYTICAL PROCEDURE

The general procedure employed follows. The thermodynamic analysis is, as a matter of convenience, based on one lb. of air delivery per cycle for each of the two compressor cylinders acting in parallel. Then, still on this unit basis, the times for the out and in-strokes are determined, and the system natural frequency of operation is established. Gas generator outlet temperature and pressure are known from the thermodynamic analysis; therefore, the necessary delivery flow rate can be determined. With the known frequency of operation and the affinity relations, the cylinder dimensions for the one lb. of air per cyl. cycle delivery can be scaled down to the required flow rate.

The design calculations are thus subdivided into the following sections:

I. Compressor Characteristics (for 1 lb of air del/cyl. cycle)

II. Engine Characteristics (for 1 lb. of air del/cyl cycle)

III. Bounce cylinder Characteristics (for 1 lb. of air del/cyl cycle)

IV. Dynamic Analysis (for 1 lb. of air del/cyl. cycle)

V. Overall Gas Generator Performance and cylinder dimensions for the required 1000 shp turbine output.

Each section consists of the following subdivisions.

A. Basic Idealizations

B. Independent Variables - numerical values

C. Basic Relations

D. Dependent Variables - numerical values

The formulae for the determination of each quantity are quoted, and the source of the corresponding numerical values is indicated by numbers in circles. Use is made of the notations of the sections by Roman Numerals and subsections by capital letters as indicated above. For Example, II D. 14 means: item 14 of Dependent Variables (D) of Engine Characteristics (II).

AFFINITY RELATIONS

By use of the affinity relationships, design results can be extrapolated for the prediction of cylinder sizes and operating frequency for an shp output maintaining the same thermodynamic idealizations and operating conditions.

Constant operating conditions are as follows:

1. State of air at the start of the compression process in the compressor, engine, and bounce cylinders,
2. Compressor pressure ratio
3. Engine compression ratio
4. Bounce cylinder compression ratio
5. Air-fuel ratio

These conditions will fix the following factors:

1. Ratio of engine bore to cylinder bore.
2. Ratio of engine air (fuel delivered and gas generated) to compressor air
3. Efficiency and fuel economy

The independent variables are:

Piston Diameter, D (The ratio of engine to compressor piston diameter is fixed as a result of maintaining the above operating conditions constant for the extrapolation, see Appendix 1-I)

2. Piston Mass, M

3. Stroke, S*

*Proportional to clearance and engine port distances in order to maintain contant operating conditions

Since Piston mass:

$$M \propto D^2 \ell \rho,$$

In stead of piston mass there can be taken, as independent variables:

2a. Piston length ℓ, and

2b. Piston bulk average density of the piston assembly.

The independence between the piston length, l, and the stroke length, s, is, to a certain extent, limited by the geometry of the engine. If stroke changes are small the piston length may remain unchanged, but if the stroke length is increased beyond a certain limit the piston length has to be increased also in order to provide enough space for the necessary piston assembly synchronizing mechanism.

Thus two groups of affinity relationships are derived. The first group corresponds to independent variations of the stroke and the piston length, and the second group refers to the stroke being changed proportionally to the piston length.

STROKE INDEPENDENT OF PISTON LENGTH

Since for the postulated constant thermodynamic operating conditions, pressures will remain the same, the resultant force on piston assembly

$$F \propto D^2$$

Thus, taking into account equation (1), the piston acceleration

$$a = \frac{F}{M} \propto \frac{D^2}{M} \propto \frac{1}{\ell \rho} \qquad \text{a - II}$$

Piston velocity will then be (See Part I equation (5-1) p. 66

$$V = \sqrt{2\int a\,dx} \;\propto\; D\sqrt{\frac{s}{M}} \;\propto \sqrt{\frac{s}{\ell\rho}} \qquad 2a - II$$

for x s.

Consequently, the operating frequency will have the following variation (see Part I equation (6-1)):

$$f_r \propto \frac{1}{T} \propto \frac{1}{\int \frac{dx}{v}} \propto \frac{D}{\sqrt{Ms}} \propto \frac{1}{\sqrt{s\ell\rho}} \qquad 3a - II$$

At constant thermodynamic operating conditions the flow per cycle is proportional to displacement, consequently

$$w_{\substack{\text{air delivered}\\\text{per cycle}}} \propto w_{\substack{\text{fuel delivered}\\\text{per cycle}}} \propto D^2 s \qquad 4a - II$$

Since the thermodynamic state of generated gas remains unchanged, the generator output:

$$hp \propto w_{gas} \propto w_{\substack{\text{air delivered}\\\text{per cycle}}} \; f_r$$

and by equations (3a) and (4a),

$$hp \propto w_{gas} \propto w_{fuel} \propto D^3\sqrt{\frac{s}{M}} \propto D^2\sqrt{\frac{s}{\ell\rho}} \qquad 5a - II$$

The overall dimensions of the gas generator are mainly dependent on the piston size. Thus, approximately, the mass of the system:

$$M_{overall} \propto D^2 \ell \rho_{overall}$$

where $\rho_{overall}$ is the bulk average density of the complete system, and the overall generator mass per unit output:

$$\frac{M_{overall}}{hp} \propto \frac{\ell\rho_{overall}}{D}\sqrt{\frac{M}{s}} \propto \ell^{3/2}\rho_{overall}\sqrt{\frac{r}{s}}$$

6a - II

STROKE PROPORTIONAL TO PISTON LENGTH

For the case of stroke proportional to piston length the affinity relationships for piston acceleration, (1a), and for the mass of air delivered per cycle (4a) remain unchanged. The affinity relationships for piston velocity, operating frequency, hp output and overall unit specific mass simplify, respectively, to the following:

$$V \propto \frac{1}{\sqrt{\rho}} \qquad 2b - II$$

$$f_r \propto \frac{1}{s\sqrt{\rho}} \propto \frac{1}{\ell\sqrt{\rho}}$$

$$hp \propto \frac{D^2}{\sqrt{\rho}}$$

and

$$\frac{M_{overall}}{hp} \propto \ell\rho_{overall}\sqrt{\rho} \propto \rho_{overall} \qquad 6b - II$$